

# The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: measurements of the growth of structure and expansion rate at $z=0.57$ from anisotropic clustering

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## ABSTRACT

We analyze the anisotropic clustering of massive galaxies from the Sloan Digital Sky Survey III Baryon Oscillation Spectroscopic Survey (BOSS) Data Release 9 (DR9) sample, which consists of 264283 galaxies in the redshift range  $0.43 < z < 0.7$  spanning 3275 square de-

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grees. Both peculiar velocities and errors in the assumed redshift-distance relation (“Alcock-Paczynski effect”) generate correlations between clustering amplitude and orientation with respect to the line-of-sight. Together with the sharp baryon acoustic oscillation (BAO) standard ruler, our measurements of the broadband shape of the monopole and quadrupole correlation functions simultaneously constrain the comoving angular diameter distance ( $2190 \pm 61$  Mpc) to  $z = 0.57$ , the Hubble expansion rate at  $z = 0.57$  ( $92.4 \pm 4.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ), and the growth rate of structure at that same redshift ( $d\sigma_8/d \ln a = 0.43 \pm 0.069$ ). Our analysis provides the best current direct determination of both  $D_A$  and  $H$  in galaxy clustering data using this technique. If we further assume a  $\Lambda$ CDM expansion history, our growth constraint tightens to  $d\sigma_8/d \ln a = 0.415 \pm 0.034$ . In combination with the cosmic microwave background, our measurements of  $D_A$ ,  $H$ , and  $d\sigma_8/d \ln a$  all separately require dark energy at  $z > 0.57$ , and when combined imply  $\Omega_\Lambda = 0.74 \pm 0.016$ , independent of the Universe’s evolution at  $z < 0.57$ . In our companion paper (Samushia et al. 2012), we explore further cosmological implications of these observations.

**Key words:** cosmology; large-scale structure of Universe, cosmological parameters, galaxies: haloes, statistics

## 1 INTRODUCTION

Measurements of the cosmic distance-redshift relation using supernovae (Riess et al. 1998; Perlmutter et al. 1999; Kessler et al. 2009; Amanullah et al. 2010), the cosmic microwave background (Larson et al. 2011), the Hubble constant (Riess et al. 2011), and baryon acoustic oscillations (BAO; Eisenstein et al. 2005; Cole et al. 2005; Hutsi 2006; Padmanabhan et al. 2007; Blake et al. 2007; Percival et al. 2007; Okumura et al. 2008; Gaztanaga, Cabre & Hui 2009; Kazin et al. 2010; Percival et al. 2010; Reid et al. 2010; Blake, et al. 2011a; Beutler et al. 2011) have revealed that the expansion of the universe is accelerating; either the energy density of the universe is dominated by an exotic “dark energy”, or general relativity requires modification. The observed anisotropic clustering of galaxies can help distinguish between these possibilities by allowing simultaneous measurements of both the geometry of the Universe and the growth rate of structure.

Galaxy redshift surveys provide a powerful measurement of the growth rate through redshift-space distortions (RSD) (Kaiser 1987). Although we expect the clustering of galaxies in real space to have no preferred direction, galaxy maps produced by estimating distances from redshifts obtained in spectroscopic surveys reveal an anisotropic galaxy distribution (Cole, Fisher, & Weinberg 1995; Peacock et al. 2001; Percival et al. 2004; da Angela et al. 2008; Okumura et al. 2008; Guzzo et al. 2008; Samushia, Percival, & Raccanelli 2011; Blake, et al. 2011b). This anisotropy arises because the recession velocities of galaxies, from which distances are inferred, include components from both the Hubble flow and from peculiar velocities driven by the clustering of matter (see Hamilton 1998 for a review). Despite the fact that galaxy light does not faithfully trace the mass, even on large scales, galaxies are expected to act nearly as test particles within the cosmological matter flow. Thus the motions of galaxies carry an imprint of the rate of growth of large-scale structure and allow us to both probe dark energy and test General Relativity (see e.g. Jain & Zhang 2008; Neseris & Perivolaropoulos 2008; Song & Koyama 2009; Song & Percival 2009; Percival & White 2008; McDonald & Seljak 2009; White, Song & Percival 2009; Song et al. 2010; Zhao et al. 2010; Song et al. 2011, for recent studies).

The observed BAO feature in the power-spectrum and correlation function of galaxies has been used to provide strong constraints on the geometry of the Universe. While the BAO method is expected to be highly robust to systematic uncertainties (see, e.g., Eisenstein & White 2004; Padmanabhan & White 2009; Seo et al. 2010; Mehta et al. 2011), it does not exploit the full information about the cosmological model available in the two-dimensional clustering of galaxies. Additional geometric information is available by comparing the amplitude of clustering along and perpendicular to the line-of-sight (LOS); this is known as the Alcock-Paczynski (AP) test (Alcock & Paczynski 1979; Ballinger, Peacock, & Heavens 1996). RSD and AP tests rely on the measurements of anisotropy in the statistical properties of the galaxy distribution and are partially degenerate with each other, so that constraints on the growth of structure from RSD depend on the assumptions about the background geometry and vice versa (Samushia et al. 2011). However, given high precision clustering measurements over a wide range of scales, this degeneracy can be broken since RSD and AP have different scale-dependences. Recently, the WiggleZ survey (Drinkwater et al. 2010) has performed a joint RSD and AP analysis that constrains the expansion history in 4 bins across  $0.1 < z < 0.9$  at the 10-15 per cent level (Blake, et al. 2011c). Using the SDSS-II LRG sample, Chuang & Wang (2011) perform a similar analysis to the present work. They measure the angular diameter distance  $D_A(z = 0.35) = 1048^{+60}_{-58}$  Mpc and the Hubble expansion rate  $H(z = 0.35) = 82.1^{+4.8}_{-4.9} \text{ km s}^{-1} \text{ Mpc}^{-1}$  at  $z = 0.35$  after marginalizing over redshift space distortions and other parameter uncertainties.

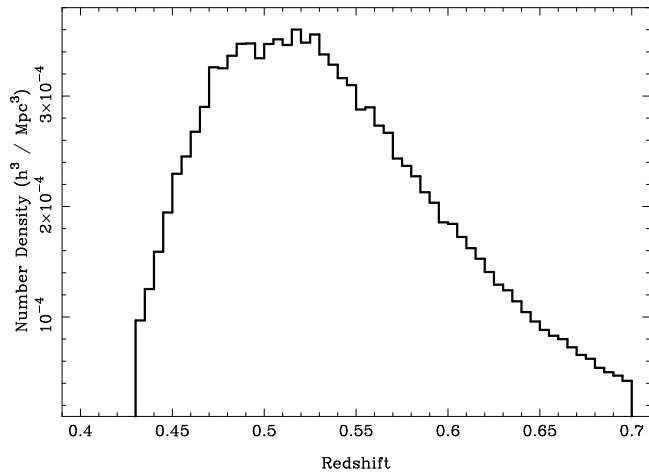
Obtaining reliable cosmological constraints from the RSD and AP effects demands precise modeling of the nonlinear evolution of both the matter density and velocity fields, as well as the ways in which the observed galaxies trace those fields. The halo model of large-scale structure and variants thereof assume that galaxies form and evolve in the potential wells of dark matter halos, and provides a successful means of parametrizing the relation between the galaxy and halo density and velocity fields. Our modeling approach, based on Reid & White (2011), uses perturbation theory to account for the nonlinear redshift space clustering of halos in the quasilinear regime as a function of cosmological parameters, and then uses the halo model framework to motivate our choice of nuisance parameters describing the galaxy-halo relation. We test these

assumptions with a large volume of mock galaxy catalogs derived from  $N$ -body simulations.

The ongoing Baryon Oscillation Spectroscopic Survey (BOSS; Schlegel, White & Eisenstein 2009), which is part of Sloan Digital Sky Survey III (SDSS-III) (Eisenstein et al. 2011), is measuring spectroscopic redshifts of 1.5 million galaxies, approximately volume limited to  $z \approx 0.6$  (in addition to spectra of 150,000 quasars and various ancillary observations). The galaxies are selected from the multi-color SDSS imaging to probe large-scale structure at intermediate redshift; they trace a large cosmological volume while having high enough number density to ensure that shot-noise is not a dominant contributor to the clustering variance (White et al. 2011). The resulting clustering measurements provide strong constraints on the parameters of standard cosmological models.

We use the CMASS sample of BOSS galaxies that will be included in SDSS Data Release 9 (DR9) to constrain the growth of structure and geometry of the Universe. We apply RSD and AP tests to the data to measure the growth rate, the Hubble expansion rate, and the comoving angular diameter distance at  $z_{\text{eff}} = 0.57$ . We improve on previous, similar analyses in a number of ways. First, we use a model for the nonlinear anisotropic correlation function that is accurate to well below our statistical errors over the wide range of scales between  $25 h^{-1}\text{Mpc}$  and  $160 h^{-1}\text{Mpc}$ , which we validate using  $68 (h^{-1}\text{Gpc})^3$  of  $N$ -body simulations populated with mock CMASS galaxies. Next, rather than assuming a fixed underlying linear matter power spectrum, we use a prior on  $P(k)$  based on the WMAP7 cosmic microwave background constraints (Larson et al. 2011; Komatsu et al. 2011) and marginalize over the remaining uncertainties for all fits. In addition to joint constraints on the geometric (BAO and AP) and peculiar velocity (RSD) parameters, we present three “null” tests of the  $\Lambda$ CDM model. To begin, we simply ask whether any points in the  $\Lambda$ CDM parameter space allowed by WMAP7 provide a good fit to the CMASS clustering; in this case, the only free parameters are those describing how galaxies trace matter. In the other two tests, we fit for the amplitude of peculiar velocities using WMAP7 priors on geometric quantities, or we fit for the geometric parameters with WMAP7 priors on RSD. Thus we can present the statistical precision with which our data measure either peculiar velocities or the AP effect in the  $\Lambda$ CDM model. Finally, given the strong detection of the BAO feature in the monopole of the correlation function, we can break the degeneracy between  $(1 + z_{\text{eff}})D_A$  and  $H$  with our AP measurement. We present the most constraining measurement of  $H(z_{\text{eff}})$  from galaxy clustering data to date using this technique, even after marginalizing over the amplitude of the RSD effect. As our constraints exploit the full shape of the monopole and quadrupole correlation functions, they rely on further assumptions about the cosmological model: Gaussian, adiabatic, power-law primordial perturbations, the standard number  $N_{\text{eff}} = 3.04$  of massless neutrino species (see discussion in Komatsu et al. 2011), and that dark matter is “cold” on the relatively large scales of interest.

This paper is organized as follows. In Sec. 2 we describe BOSS DR9 CMASS data and in Sec. 3 we describe the measurements of two-dimensional clustering of galaxies used in this analysis. Sec. 4 reviews the theory of the RSD and AP effects, and describes the theoretical model used to fit our measurements. Sec. 5 outlines the methods we use to analyse the data and Sec. 6 presents the results of analysis. We conclude by discussing the cosmological implications of our results in Sec. 7.



**Figure 1.** Number density as a function of redshift for the CMASS galaxies used in this analysis. After accounting for our weighting scheme, the effective redshift for galaxy pairs in this sample is  $z_{\text{eff}} = 0.57$ .

## 2 DATA

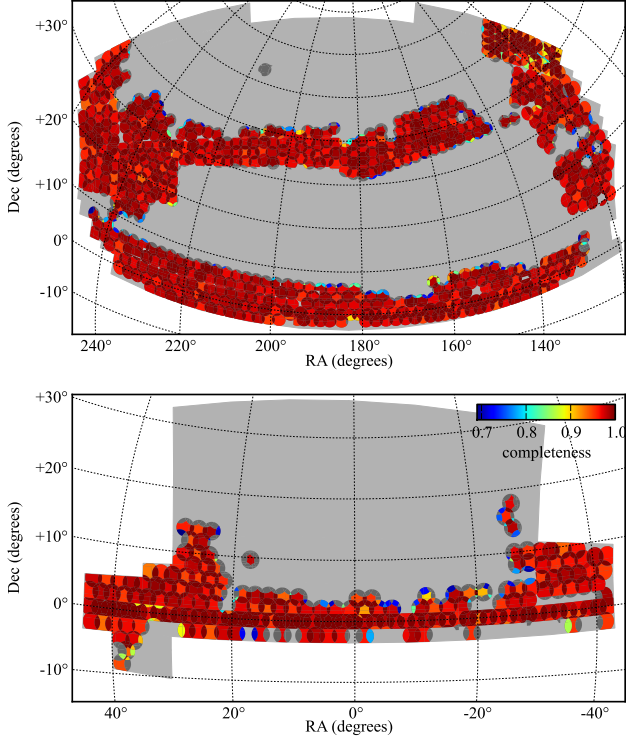
BOSS targets for spectroscopy luminous galaxies selected from the multi-color SDSS imaging (Fukugita et al. 1996; Gunn et al. 1998; York et al. 2000; Gunn et al. 2006). The target selection algorithms are summarized by Eisenstein et al. (2011) and Anderson et al. (2012). For the galaxy sample referred to as “CMASS”, color-magnitude cuts are applied to select a roughly volume-limited sample of massive, luminous galaxies; see Masters et al. (2011) for a detailed examination of the properties of CMASS targets in the COSMOS field. The majority of the galaxies are old stellar systems whose prominent 4000 Å break in their spectral energy distributions makes them relatively easy to select in multi-color data. Most CMASS galaxies are central galaxies residing in dark matter halos of  $10^{13} h^{-1} M_{\odot}$ , but a non-negligible fraction are satellites which live primarily in halos about 10 times more massive (White et al. 2011). These galaxies are intrinsically very luminous and at the high mass end of the stellar mass function (Maraston et al. prep). Galaxies in the CMASS sample are highly biased ( $b \sim 2$ , White et al. 2011). In addition, they trace a large cosmological volume while having high enough number density to ensure that shot-noise is not a dominant contributor to the clustering variance, which makes them particularly powerful for probing statistical properties of large-scale structure.

Anderson et al. (2012) details the steps for generating the large-scale structure catalog and mask for DR9, which includes the data taken by BOSS through July 2011 and covers  $3275 \text{ deg}^2$  of sky. In our analysis we use galaxies from the BOSS CMASS DR9 catalog in the redshift range of  $0.43 < z < 0.70$ . The sample includes a total of 264,283 galaxies, 207,246 in the north and 57,037 in the south Galactic hemispheres. Figure 1 shows the redshift distribution and Figure 2 shows the angular distribution of the galaxies in our sample.

## 3 MEASUREMENTS

### 3.1 Two point statistics

To compute redshift space separations for each pair of galaxies given their angular coordinates and redshifts, we must adopt a cosmological model. We use the same one as used to generate our



**Figure 2.** BOSS DR9 sky coverage. The light gray region shows the expected total footprint of the survey, while the colors indicate completeness in the DR9 spectroscopy for regions included in our analysis. Dark grey regions are removed from the analysis by completeness or redshift failure cuts; see Sec. 3.5 of Anderson et al. (2012) for further details.

mock catalogs, namely spatially-flat  $\Lambda$ CDM with  $\Omega_m = 0.274$ . This is the same cosmology as assumed in White et al. (2011) and in our companion papers (Anderson et al. 2012; Manera et al. 2012; Ross et al. 2012; Sanchez et al. 2012; Samushia et al. 2012; Tojeiro et al. 2012). Our model accounts for this assumption and scales the theory prediction accordingly when testing a cosmological model with a different distance-redshift relation; see Sec. 4.5.

Using the galaxy catalog of Anderson et al. (2012) we compute weighted “data-data” (DD) pair counts in bins of  $s$  and  $\mu$

$$DD(s_i, \mu_j) = \sum_{k=1}^{N_{\text{tot}}} \sum_{l=k+1}^{N_{\text{tot}}} \Theta_{k,l}(s_i, \mu_j) w_k(s_i) w_l(s_i), \quad (1)$$

where  $s$  is the comoving pair separation in redshift space,  $\mu$  is the cosine of the angle between the pair separation vector and the LOS and  $w_k$  is the weight of  $k^{\text{th}}$  galaxy in the catalog. The double sum runs over all galaxies and  $\Theta_{k,l}(s_i, \mu_j) = 1$  if the pair separation between two galaxies falls into bin  $s_i, \mu_j$ , and is zero otherwise.

Three distinct effects contribute to the final weight  $w_i$  of each galaxy. These weights are described in more detail in Anderson et al. (2012) and Ross et al. (2012). First, galaxies lacking a redshift due to fiber collisions or because their spectrum was not adequate to secure a redshift are accounted for by upweighting the nearest galaxy by weight  $w = (1 + n)$ , where  $n$  is the number of near neighbors without a redshift. Second, we use the minimum variance weighting of Hamilton (1993),

$$w(s) = \frac{1}{1 + J_3(s) \bar{n}(z)}, \quad (2)$$

where  $\bar{n}(z)$  is the expected number density of galaxies at the redshift

of the galaxy and

$$J_3(s) = 2\pi \int_0^s s'^2 ds' d\mu \xi(s', \mu) \quad (3)$$

is the angularly averaged redshift-space correlation function integrated up to the separation of galaxies in the pair. For every galaxy this weight will vary depending on which pair-counting bin it is assigned. For a constant radial selection function this weighting scheme results in the minimal variance of the estimated correlation function (for details see Hamilton 1993). Note that the analyses in Sanchez et al. (2012) and Anderson et al. (2012) use scale-independent weights; differences between the approaches are small in practice.

The third weight corrects for angular systematics, related to the angular variations in density of stars that make detection of galaxies harder in areas of sky closer to the Galactic equator (for details see Ross et al. 2012). The total weight is the product of these three weights. We bin  $s$  in 23 equal logarithmic bins between  $s_{\text{min}} = 25.1$  and  $s_{\text{max}} = 160 h^{-1} \text{Mpc}$  with  $d\log_{10}s = 0.035$ , and 200 equally-spaced  $\mu$  bins between 0 and 1. We compute “data-random” (DR) and “random-random” (RR) pair counts as in Eqn. 1, except that each random point is assigned only the  $J_3(s)$  weight and not the close-pair correction and systematic weights. Positions of objects in our random catalog are generated using the observational mask and redshifts are generated by picking a random redshift drawn from the measured redshifts of observed galaxies. Our random catalogs contain approximately 70 times more objects than the galaxy catalog.

Following Landy & Szalay (1993), the pair counts are combined to estimate the anisotropic correlation function as:

$$\hat{\xi}_{LS}(s_i, \mu_j) = \frac{DD(s_i, \mu_j) - 2DR(s_i, \mu_j) + RR(s_i, \mu_j)}{RR(s_i, \mu_j)}. \quad (4)$$

Figure 3 shows our measurement of  $\hat{\xi}_{LS}(s_i, \mu_j)$  in terms of LOS separation  $r_\pi = s\mu_s$  and transverse separation  $r_\sigma = s(1 - \mu^2)^{1/2}$ . The central “squashing” is due to peculiar velocities. In the left panel, the BAO ridge at  $\sim 100 h^{-1} \text{Mpc}$  is evident. In the right panel, we show the clustering signal on smaller scales; the “finger-of-God” effect is visible for small transverse separations but small on the scales we analyse. The innermost contour in the right panel indicates the value of  $\xi_0$  in the smallest separation bin included in our cosmological analysis.

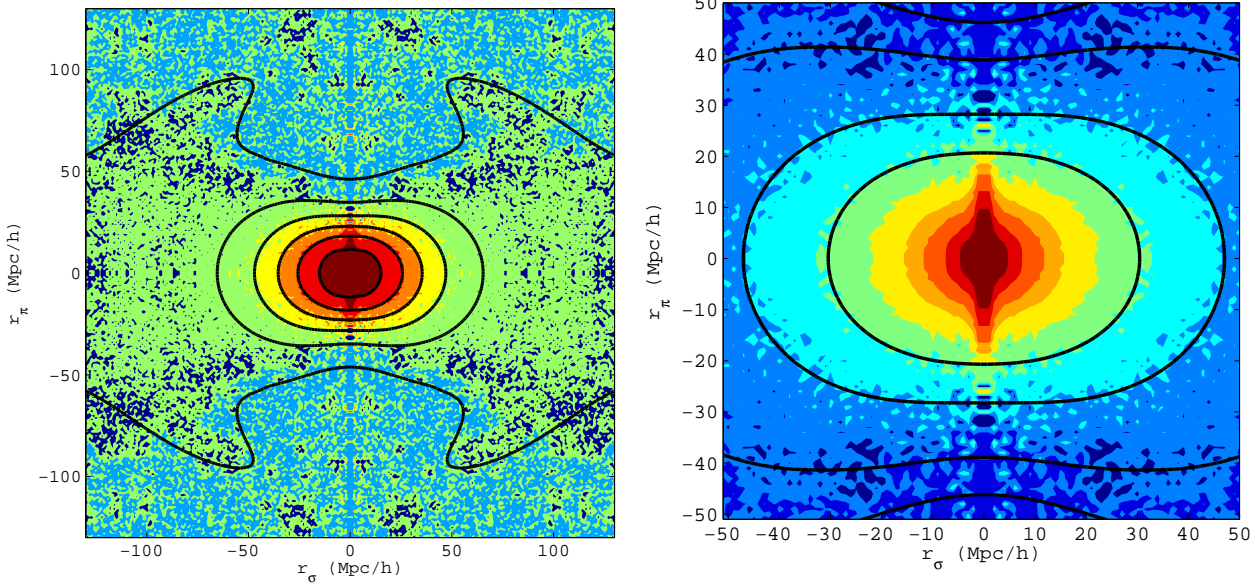
Rather than work with the two-dimensional correlation function  $\xi(s, \mu_s)$ , we conduct our cosmological analysis on the first two even Legendre polynomial moments,  $\xi_0(s)$  and  $\xi_2(s)$ , defined by

$$\xi_\ell(s) = \frac{2\ell + 1}{2} \int d\mu_s \xi(s, \mu_s) L_\ell(\mu_s), \quad (5)$$

or equivalently,

$$\xi(s, \mu_s) \equiv \sum_{\ell=0}^{\infty} \xi_\ell(s) L_\ell(\mu_s). \quad (6)$$

Here  $L_\ell$  is the Legendre polynomial of order  $\ell$ . By symmetry all odd- $\ell$  moments vanish and on large scales the measurements become increasingly noisy to larger  $\ell$ . The correlation functions  $\hat{\xi}_0(s)$  and  $\hat{\xi}_2(s)$  are estimated from  $\hat{\xi}_{LS}(s_i, \mu_j)$  using a Riemann sum to approximate Eqn. 5. We include all galaxy pairs between 25 and  $160 h^{-1} \text{Mpc}$  in our analysis. We also caution the reader that we have adopted logarithmically-spaced bins, while our companion papers (Anderson et al. 2012; Ross et al. 2012; Sanchez et al. 2012) analyze clustering in linearly-spaced bins of differing bin sizes. Our measurements of  $\xi_0$  and  $\xi_2$ , along with diagonal errors estimated from mock catalogs (Manera et al. 2012; see Sec. 3.2) are shown



**Figure 3.** *Left panel:* Two-dimensional correlation function of CMASS galaxies (color) compared with the best fit model described in Section 6.1 (black lines). Contours of equal  $\xi$  are shown at [0.6, 0.2, 0.1, 0.05, 0.02, 0]. *Right panel:* Smaller-scale two-dimensional clustering. We show model contours at [0.14, 0.05, 0.01, 0]. The value of  $\xi_0$  at the minimum separation bin in our analysis is shown as the innermost contour. The  $\mu \approx 1$  “finger-of-god” effects are small on the scales we use in this analysis.

in Figure 4. The effective redshift of weighted pairs of galaxies in our sample is  $z = 0.57$ , with negligible scale dependence for the range of interest in this paper. For the purposes of constraining cosmological models, we will interpret our measurements as being at  $z = 0.57$ .

### 3.2 Covariance Matrices

The matrix describing the expected covariance of our measurements of  $\xi_\ell(s)$  in bins of redshift space separation depends in linear theory only on the underlying linear matter power spectrum, the bias of the galaxies, the shot-noise (often assumed Poisson) and the geometry of the survey. We use 600 mock galaxy catalogs, based on Lagrangian perturbation theory (LPT) and described in detail in Manera et al. (2012), to estimate the covariance matrix of our measurements. We compute  $\xi_\ell(s_i)$  for each mock in exactly the same way as from the data (Sec. 3.1) and estimate the covariance matrix as

$$C_{ij}^{\ell_1 \ell_2} = \frac{1}{599} \sum_{k=1}^{600} \left( \xi_{\ell_1}^k(s_i) - \bar{\xi}_{\ell_1}(s_i) \right) \left( \xi_{\ell_2}^k(s_j) - \bar{\xi}_{\ell_2}(s_j) \right), \quad (7)$$

where  $\xi_\ell^k(s_i)$  is the monopole ( $\ell = 0$ ) or quadrupole ( $\ell = 2$ ) correlation function for pairs in the  $i^{\text{th}}$  separation bin in the  $k^{\text{th}}$  mock.  $\bar{\xi}_\ell(s)$  is the mean value over all 600 mocks. The shape and amplitude of the average two-dimensional correlation function computed from the mocks is a good match to the measured correlation function of the CMASS galaxies; see Manera et al. (2012) and Ross et al. (2012) for more detailed comparisons. The square roots of the diagonal elements of our covariance matrix are shown as the error bars accompanying our measurements in Fig. 4. We will examine the off-diagonal terms in the covariance matrix via the correlation

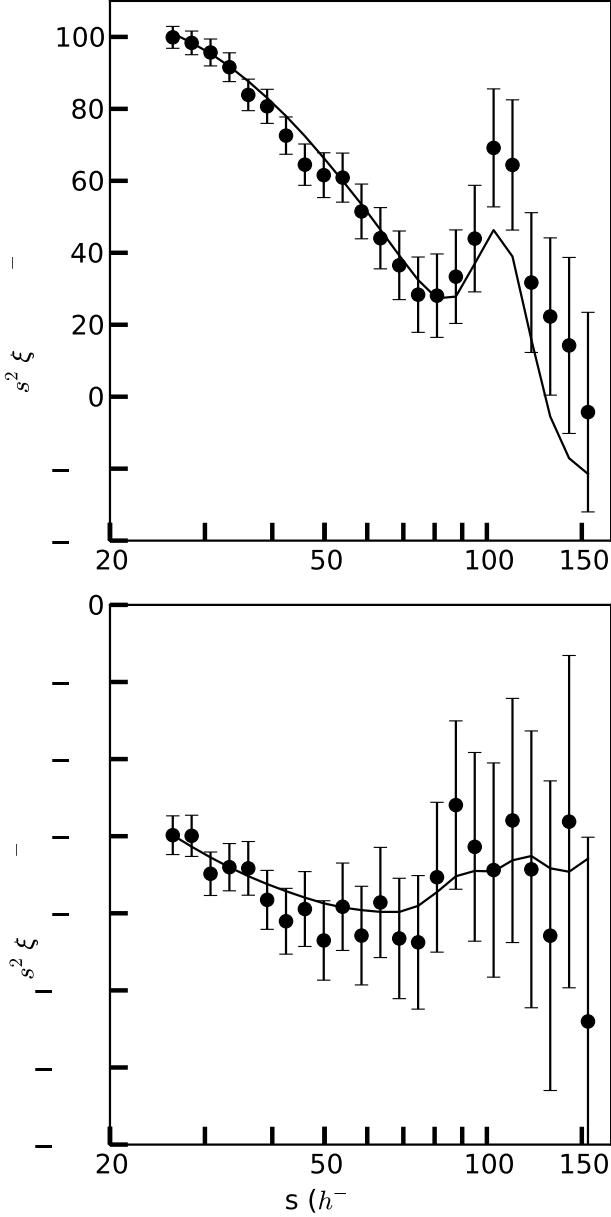
matrix, or “reduced covariance matrix”, defined as

$$C_{ij}^{\ell_1 \ell_2, \text{red}} = C_{ij}^{\ell_1 \ell_2} / \sqrt{C_{ii}^{\ell_1 \ell_1} C_{jj}^{\ell_2 \ell_2}}, \quad (8)$$

where the division sign denotes a term by term division.

In Figure 5 we compare selected slices of our mock covariance matrix (points) to a simplified prediction from linear theory (solid lines) that assumes a constant number density  $\bar{n} = 3 \times 10^{-4} (h^{-1} \text{ Mpc})^{-3}$  and neglects the effects of survey geometry (see, e.g., Tegmark 1997). Xu et al. (2012) performed a detailed comparison of linear theory predictions with measurements from the Las Damas SDSS-II LRG mock catalogs (McBride et al. prep), and showed that a modified version of the linear theory covariance with a few extra parameters provides a good description of the  $N$ -body based covariances for  $\xi_0(s)$ . The same seems to be true here as well. The mock catalogs show a deviation from the naive linear theory prediction for  $\xi_2(s)$  on small scales; a direct consequence is that our errors on quantities dependent on the quadrupole are larger than a simple Fisher analysis would indicate. We verify that the same qualitative behavior is seen for the diagonal elements of the quadrupole covariance matrix in our smaller set of  $N$ -body simulations used to calibrate the model correlation function. This comparison suggests that the LPT-based mocks are not underestimating the errors on  $\xi_2$ , though more  $N$ -body simulations (and an accounting of survey geometry) would be required for a detailed check of the LPT-based mocks.

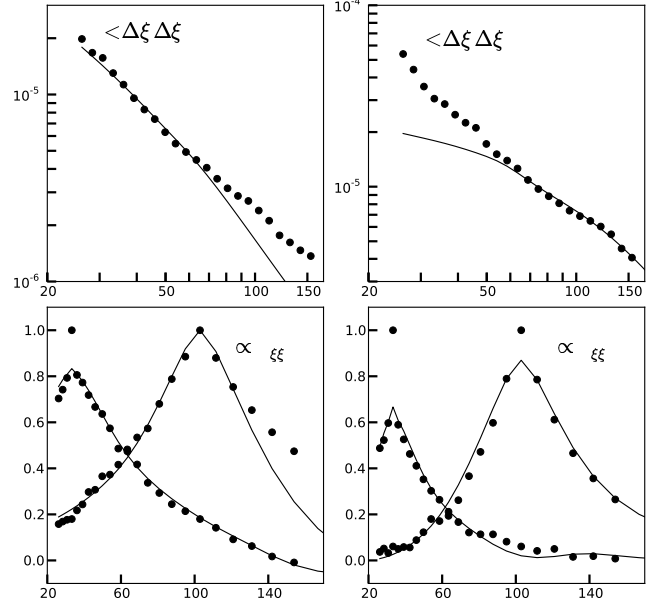
The lower panels of Figure 5 compare the reduced covariance matrix to linear theory, where we have scaled the  $C_{ij}^{\text{red}}$  prediction from linear theory down by a constant,  $c_\ell$ . This comparison demonstrates that the scale dependences of the off-diagonal terms in the covariance matrix are described well by linear theory, but that the nonlinear evolution captured by the LPT mocks can be parametrized simply as an additional diagonal term. Finally,



**Figure 4.**  $\xi_0(s)$  and  $\xi_2(s)$  measured from BOSS CMASS galaxies. The errorbars correspond to diagonal elements of the covariance matrix. The best-fitting model described in Section 6.1 is shown as the solid curve. We use 23 logarithmically spaced bins and include pairs between 25 and 160  $h^{-1}$  Mpc.

while not shown here, the reduced covariances between  $\xi_0$  and  $\xi_2$  are small.

Our analysis uses the LPT mock-based covariance matrix, which accurately accounts for both complexities of the survey geometry as well as nonlinear corrections to the growth of structure on the relatively large scales of interest here, and this allows us to accurately report uncertainties associated with both our measurements and parameter fits.



**Figure 5.** *Upper panels:* Diagonal elements of the monopole and quadrupole components of the covariance matrix computed from the LPT-based mocks (points) compared with the linear theory prediction (solid lines). *Lower panels:* Two slices through the reduced covariance matrices  $C_{ij}^{00,\text{red}}$  and  $C_{ij}^{22,\text{red}}$  for separation bins of 33 and 103  $h^{-1}$  Mpc. Linear theory predictions for the reduced covariance matrices in the lower panels have been scaled by a constant factor to produce good agreement between linear theory and mock covariances for off-diagonal elements, demonstrating that the scale dependence of the off-diagonal terms matches the mock covariance matrix well, but that there is extra diagonal covariance in the mocks compared with linear theory. Elements of  $C^{02}$  are small (not shown).

## 4 THEORY

### 4.1 Redshift Space Distortions: Linear Theory

The effects of redshift space distortions in the linear regime are well-known (Kaiser 1987; Fisher 1995, see also Hamilton 1998 for a comprehensive review). We briefly summarize them here. The redshift-space position,  $\mathbf{s}$ , of a galaxy differs from its real-space position,  $\mathbf{x}$ , due to its peculiar velocity,

$$\mathbf{s} = \mathbf{x} + v_z(\mathbf{x}) \hat{\mathbf{z}}, \quad (9)$$

where  $v_z(\mathbf{x}) \equiv u_z(\mathbf{x})/(aH)$  is the change in the apparent LOS position of a galaxy due to the contribution of the LOS peculiar velocity  $u_z$  to the galaxy's redshift. Since overdensities on large, linear scales grow in a converging velocity field ( $\nabla \cdot \mathbf{v} = -f\delta_m$ ), the effect of peculiar velocities induces a coherent distortion in the measured clustering of galaxies that allows us to measure the amplitude of the peculiar velocity field. In linear theory, and with some approximations, the anisotropic galaxy power spectrum becomes (Kaiser 1987)

$$P_g^s(k, \mu_k) = (b + f\mu_k^2)^2 P_m^r(k) = b^2 (1 + \beta\mu_k^2)^2 P_m^r(k) \quad (10)$$

where  $b$  is the linear galaxy bias,  $\delta_g = b\delta_m$ ,  $f \equiv d \ln \sigma_8 / d \ln a$  is the logarithmic growth rate of matter fluctuations, and  $\mu_k$  is the cosine of the angle between  $\mathbf{k}$  and the LOS.



## 4.2 Legendre Moments of $\xi(\mathbf{r})$

In linear theory (Eqn. 10), only the  $\ell = 0, 2$  and 4 moments contribute to the power spectrum  $P_g^s(\mathbf{k})$ , and its Fourier transform  $\xi_g^s(\mathbf{s})$ . The two are simply related by  $\ell^{\text{th}}$  order Bessel functions:

$$\xi_\ell(s) = i^\ell \int \frac{k^2 dk}{2\pi^2} P_\ell(k) j_\ell(ks). \quad (11)$$

Given a tight constraint on the underlying *shape* of the linear matter power spectrum, the two-dimensional clustering of galaxies constrains both  $b\sigma_8$  and  $f\sigma_8$  (Percival & White 2008; White, Song & Percival 2009). In this work we measure and model only the monopole and quadrupole moments of the correlation function,  $\xi_{0,2}(s)$ . These two moments are sufficient to constrain both  $b\sigma_8$  and  $f\sigma_8$  and encompass most of the available information on the peculiar velocity field for the highly biased galaxies of interest here (Reid & White 2011), in addition to being more easily modelled than higher- $\ell$  moments (but see also Taruya, Saito, & Nishimichi 2011). Therefore, we collapse the anisotropic clustering information in  $\xi(r_\sigma, r_\pi)$  into two one-dimensional functions  $\xi_{0,2}(s)$  before extracting cosmological information from the anisotropic galaxy clustering.

## 4.3 Redshift space halo clustering in the quasilinear regime

The Kaiser formula describing the linear effect of redshift space distortions breaks down even on quite large scales. An accurate model of the two-dimensional clustering of galaxies must account for nonlinear evolution in the real space matter density and velocity fields, nonlinear galaxy bias, and the nonlinear mapping between real and redshift space separations of pairs of galaxies. The simplest picture of galaxy formation asserts that galaxies occupy dark matter halos, and so as a step towards understanding the clustering of galaxies, Reid & White (2011) showed that a streaming model where the pairwise velocity probability distribution function is approximated as Gaussian can be used to relate real space clustering and pairwise velocity statistics of halos to their clustering in redshift space. We will demonstrate in Sec. 4.4 that the same model describes the clustering of galaxies:

$$1 + \xi_g^s(r_\sigma, r_\pi) = \int \left[ 1 + \xi_g^r(r) \right] e^{-[r_\pi - y - \mu v_{12}(r)]^2 / 2\sigma_{12}^2(r, \mu)} \frac{dy}{\sqrt{2\pi\sigma_{12}^2(r, \mu)}}, \quad (12)$$

where  $r_\sigma$  and  $r_\pi$  are the redshift space transverse and LOS distances between two objects with respect to the observer,  $y$  is the *real* space LOS pair separation,  $\mu = y/r$ ,  $\xi_g^r$  is the real space galaxy correlation function,  $v_{12}(r)$  is the average infall velocity of galaxies separated by real-space distance  $r$ , and  $\sigma_{12}^2(r, \mu)$  is the rms dispersion of the pairwise velocity between two galaxies separated with transverse (LOS) real space separation  $r_\sigma$  ( $y$ ).

$\xi_g^r(r)$ ,  $v_{12}(r)$  and  $\sigma_{12}^2(r, \mu)$  are computed in the framework of Lagrangian ( $\xi^r$ ) and standard perturbation theories ( $v_{12}$ ,  $\sigma_{12}^2$ ). Only two nuisance parameters are necessary to describe the clustering of a sample of halos or galaxies in this model:  $b_{1L} = b - 1$ , the first-order Lagrangian host halo bias in *real* space, and  $\sigma_{\text{FoG}}^2$ , an additive, isotropic velocity dispersion accounting for small-scale motions of halos and galaxies which will be described below. Further details of the model, its numerical implementation, and its accuracy can be found in Reid & White (2011) and Appendix B.

## 4.4 From halos to galaxies

Reid & White (2011) examined the validity of Eqn. 12 only for

halo clustering rather than galaxies, thus the model must be extended and checked with a realistic sample of mock galaxies. We use the machinery of the halo model (see Cooray & Sheth 2002, for a review) to describe the galaxy density field in terms of the density field of the host halos. Of particular importance for modeling redshift space distortions is the distinction between “central” and “satellite” galaxies (Kauffmann, White, & Guiderdoni 1993; Cole et al. 1994; Kravtsov et al. 2004). When modeling an approximately mass-limited galaxy sample, the first galaxy assigned to a host halo is considered central, and its position and velocity are that of the host halo center. Satellite galaxies orbit in the potential well of the host halos, and so are offset in both position and velocity from the halo center. In our particular implementation of the halo model, satellite galaxies are randomly drawn from the dark matter particle members of the host halo in our simulation. As the virial velocities of massive halos can be large (amounting to redshift space LOS separations of tens of  $h^{-1}$  Mpc), intrahalo velocities can distort the redshift space correlation function. In the limit that these virial motions are uncorrelated with the quasilinear velocity field of interest, they can be accounted for by additional convolution along the line of sight.

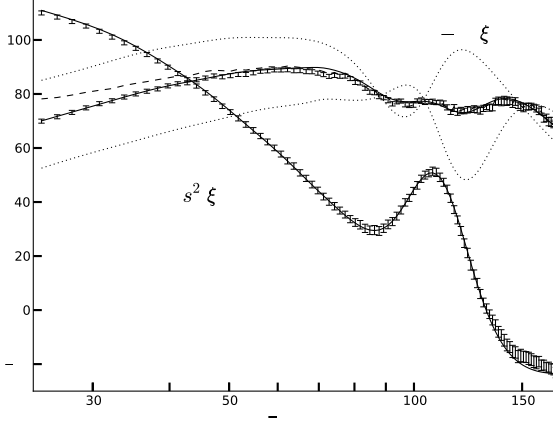
To assess the impact of virial motions on the observed galaxy clustering we use the mock catalogs described in White et al. (2011), which closely match the small-scale clustering of CMASS galaxies. We compute  $\xi_{0,2}(s)$  from these mocks (the average is shown as the error bars in Figure 6) and recompute  $\xi_{0,2}$  after artificially setting the intrahalo velocities to 0 (the dashed curves, with the one for  $\xi_0$  covered by the solid line). Intrahalo velocities suppress the amplitude of  $\xi_2$  on the smallest scales we attempt to model, reaching a 10 per cent correction at  $25 h^{-1}$  Mpc. The reason for this suppression is that, on small scales,  $d\xi^r/dy < 0$  and non-negligible. This causes a net transport of pairs to larger separations in redshift space. Note this is opposite to the effect of quasilinear peculiar velocities, which make the separation of a pair in redshift space on average smaller than in real space.

We include the effect of intrahalo velocities in our model by including an extra convolution with a Gaussian of dispersion  $\sigma_{\text{FoG}}$ . The solid curves in Fig. 6 are then the predictions for  $\xi_{0,2}$  with the best fit  $\sigma_{\text{FoG}}^2 = (3.2 h^{-1} \text{Mpc})^2$  or  $21 \text{Mpc}^2$ . The model successfully describes the effect of intrahalo velocities on the monopole and quadrupole correlation functions.

In Appendix B4 we quantify the impact of our uncertainties in the halo occupation distribution (HOD) of CMASS galaxies and the possible breakdown of our assumption that the first galaxy assigned to each halo is “central” (i.e., has no intrahalo velocity dispersion) on the value of our nuisance parameter  $\sigma_{\text{FoG}}^2$ . As a result of these considerations, we place a uniform prior on  $\sigma_{\text{FoG}}^2$  between 0 and  $40 (\text{Mpc})^2$ .

## 4.5 Alcock-Paczynski Effect

Galaxy redshift surveys collect two angular coordinates and a redshift for each galaxy in the sample. A fiducial cosmological model must be adopted to generate maps and measure clustering as a function of comoving separations. This mapping depends on both the angular diameter distance and the inverse of the Hubble parameter at the redshift of each galaxy pair. To a good approximation the inferred galaxy clustering in a different cosmological model can be obtained from the fiducial one by a single rescaling of the transverse and parallel separations (Percival et al. 2010). Rather than modify our observed galaxy clustering, we will account for the AP effect when we test different cosmological models by introducing



**Figure 6.** Error bars enclose the mean  $s^2 \xi_{0.2}(s)$  measured from the White et al. (2011) mock galaxy catalogs. The solid line is our model fit, where  $\sigma_{\text{FoG}}^2$  has been varied to minimize the difference. The dashed line shows  $s^2 \xi_2$  for the mock galaxies when their intrahalo velocities are artificially set to 0, and indicates that intrahalo velocities suppress  $\xi_2$  by  $\approx 10$  per cent on the smallest scales we are fitting. The nuisance parameter  $\sigma_{\text{FoG}}^2$  adequately describes the effect of intrahalo velocities. Dotted lines show the predicted  $\xi_2$  when varying the Alcock-Paczynski parameter  $F(z)$  by  $\pm 10$  per cent and holding  $D_V$  (and thus  $\xi_0$ ) fixed.

two scale parameters,  $\alpha_\perp$  and  $\alpha_\parallel$ , into the theoretical correlation functions we are fitting to:

$$\xi_{\text{fid}}(r_\sigma, r_\pi) = \xi^{\text{true}}(\alpha_\perp r_\sigma, \alpha_\parallel r_\pi), \quad (13)$$

$$\alpha_\perp = \frac{D_A^{\text{fid}}(z_{\text{eff}})}{D_A^{\text{true}}(z_{\text{eff}})}, \quad \alpha_\parallel = \frac{H^{\text{true}}(z_{\text{eff}})}{H^{\text{fid}}(z_{\text{eff}})}, \quad (14)$$

where  $D_A$  denotes the physical angular diameter distance. Here  $\xi^{\text{true}}$  is the expected two-dimensional correlation function if the measured galaxy correlation function were computed assuming the true redshift-distance relation in the cosmology being tested.  $\xi^{\text{fid}}$  is the prediction for the measured correlation function, given that galaxy separations were computed using the fiducial cosmology model. That is,  $\alpha_\perp$  and  $\alpha_\parallel$  scale the ‘true’ separations to the ones calculated using the fiducial cosmology.

The spherically averaged correlation function,  $\xi_0$ , is sensitive to the parameter combination

$$D_V(z) \equiv \left( (1+z)^2 D_A^2(z) \frac{cz}{H(z)} \right)^{1/3}. \quad (15)$$

The quadrupole of the measured correlation function allows a measurement of a second combination (Alcock & Paczynski 1979; Padmanabhan & White 2008; Blake, et al. 2011c)

$$F(z) = (1+z)D_A(z)H(z)/c, \quad (16)$$

thus breaking the degeneracy between  $(1+z_{\text{eff}})D_A$  and  $H$ . To a good approximation, changing  $D_V$  simply rescales the value of  $s$  in the predicted correlation function, while  $F(z)$  primarily affects the quadrupole. Figure 6 shows the effect of varying  $F$  by  $\pm 10$  per cent on  $\xi_2$  at fixed  $D_V$  with the dotted curves. The scale-dependence of  $\Delta \xi_2$  due to the AP effect will allow us to separate the effects of peculiar velocities and the AP effect.

## 5 ANALYSIS

### 5.1 Cosmological Model Space

Given an underlying linear matter power spectrum shape  $P_{\text{lin}}(k, z_{\text{eff}})$ , we consider the predicted galaxy clustering as a function of five parameters:  $\vec{p}_{\text{gal}} = \{b\sigma_8, f\sigma_8, \sigma_{\text{FoG}}, (1+z_{\text{eff}})D_A(z_{\text{eff}}), H(z_{\text{eff}})\}$ . Since the normalization of  $P_{\text{lin}}(k, z_{\text{eff}})$  (denoted throughout as  $\sigma_8$ ) determines the amplitude of the second-order perturbation theory corrections in our model, in principle we should be able to separately determine  $b\sigma_8$ ,  $f\sigma_8$ , and  $\sigma_8$ . In practice, the dependence is sufficiently small and degenerate with the nuisance parameters  $b\sigma_8$  and  $\sigma_{\text{FoG}}^2$  that the degeneracy cannot be broken; see Appendix B3 for details.

We have also assumed that any error in the fiducial cosmological model used to compute  $\xi_{0.2}(s)$  from the CMASS galaxy catalog can be absorbed in a single scaling of distances, interpreted at the effective redshift of the survey. If the assumed redshift dependence of  $(1+z_{\text{eff}})D_A(z)$  and  $H(z)$  is grossly incorrect, we would expect a difference in the correlation functions split on redshift; we see no evidence for this in our tests (Ross et al. 2012).

#### 5.1.1 Prior on the linear matter power spectrum from WMAP7

While the scale dependence of galaxy clustering itself can constrain the shape of the linear matter power spectrum, at present (and certainly with the imminent public release of Planck data) the constraints enabled by CMB measurements are stronger. The strong CMB constraints mean we can use the entire linear matter power spectrum as a standard ruler determined by observations of the CMB, rather than only the BAO feature. This approach relies on further cosmological model assumptions that are consistent with the current data, but from which moderate deviations are still allowed.

The temperature of the CMB has been measured exquisitely well (Mather et al. 1994), and determines the physical energy density in radiation,  $\Omega_r h^2$ . In the minimal cosmological model allowed by current observations (Larson et al. 2011), namely a flat  $\Lambda$ CDM cosmology with nearly scale-invariant scalar, adiabatic, Gaussian fluctuations along with the three standard, nearly massless neutrino species, only three additional parameters determine the shape of the underlying linear matter power spectrum,  $P_{\text{lin}}(k)$ . Relative peak heights in the CMB determine the physical energy densities in cold and baryonic matter,  $\Omega_{c,b} h^2$ , and the overall scale-dependence of the CMB power spectrum determines the spectral index  $n_s$  of the nearly scale-invariant scalar primordial fluctuations. Constraints on these parameters do not rely on the distance to the cosmic microwave background, and thus are immune to the behavior of dark energy at lower redshifts than the last scattering surface. Moreover, small-scale CMB experiments (Keisler et al. 2011; Hlozek et al. 2011) now probe fluctuations on the same scales as galaxy clustering measurements, and find no compelling evidence for, e.g., a running of the spectral index. Allowing for running of the spectral index would degrade the CMB constraints on the linear matter power spectrum (e.g., Mehta et al. 2012) but we will not include this additional parameter—obviously, our constraints should be interpreted in the context of our model assumptions.

One important extension of the minimal cosmological model is allowing neutrinos to have mass; neutrino oscillation experiments suggest that  $\sum m_\nu \gtrsim 0.05$  eV (Abazajian et al. 2011). As the universe expands and cools, massive neutrinos become non-relativistic and modify the linear matter power spectrum inferred from the



CMB, as well as alter the expansion history as compared with the massless neutrino case. In this work we ignore this additional uncertainty in the shape of  $P_{\text{lin}}(k)$ . Current upper bounds that combine several cosmological probes find  $\sum m_\nu \lesssim 0.3$  eV (e.g., Reid et al. 2010; de Putter et al. 2012), which is safely below the detectable level in the DR9 CMASS sample (Sanchez et al. 2012).

As we explore the cosmological constraints available from our data set, we will consider a set of power-spectrum shapes parametrized by  $\vec{p}_s = \{\Omega_b h^2, \Omega_c h^2, n_s\}$ . We will marginalize over parameters  $\vec{p}_s$  either by importance resampling (Gilks, Richardson & Spiegelhalter 1996; Lewis & Bridle 2002) the public Monte Carlo Markov Chains for a spatially flat  $\Lambda$ CDM model provided by the WMAP collaboration, or by approximating the constraints on  $\vec{p}_s$  with a three-dimensional Gaussian likelihood. Note that the three-dimensional constraints on linear matter power spectrum parameters are very nearly independent of cosmological model extensions that change the expansion rate only for  $z \ll 1000$  (i.e., introducing  $w$  or  $\Omega_k$ ).

## 5.2 Models

In the present paper, we interpret the anisotropic clustering of CMASS galaxies only in the context of the  $\Lambda$ CDM cosmology. By relaxing assumptions about the redshift-distance relation and/or the growth of structure in the  $\Lambda$ CDM model, we report the statistical precision with which CMASS measurements constrain both the peculiar velocity field  $f\sigma_8(z_{\text{eff}})$  and geometric quantities  $D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$ , without adopting a particular cosmological model that specifies how these quantities are related. We consider the following four models:

- **Model 1: WMAP7+CMASS flat  $\Lambda$ CDM.** In the flat  $\Lambda$ CDM cosmology,  $f\sigma_8(z_{\text{eff}})$ ,  $D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  are all determined once  $\Omega_m$ ,  $\sigma_8$ , and  $H_0$  are specified. This provides a null test of our assumptions relating CMB fluctuations to predicted galaxy fluctuations.

- **Model 2 WMAP7+CMASS  $\Lambda$ CDM geometry, free growth.** The analysis for this model is the same as Model 1, except that we consider  $f\sigma_8$  a free parameter in the CMASS galaxy clustering fits.

- **Model 3 WMAP7+CMASS  $\Lambda$ CDM growth, free geometry** The analysis for this model is the same as Model 1, except that we consider  $D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  as free parameters in the CMASS galaxy clustering fits.

- **Model 4 WMAP7+CMASS, free growth, free geometry:** In this model,  $f\sigma_8(z_{\text{eff}})$ ,  $D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  are all free parameters in the fit to the galaxy clustering data. This allows us to determine how well our data break the degeneracy between the RSD and AP effects, and to present the constraints originating from the amplitude and scale-dependence of the galaxy quadrupole  $\xi_2(s)$  in the most model independent way possible. This multivariate distribution can be used to constrain any model that does not alter the shape of the linear matter power spectrum at  $z_{\text{eff}}$  from that inferred from the CMB on the relatively large scales of interest here.

In all four scenarios we allow the value of  $\sigma_{\text{FoG}}^2$  to vary between 0  $\text{Mpc}^2$  and 40  $\text{Mpc}^2$  with a flat prior and marginalize over both  $\sigma_{\text{FoG}}^2$  and  $b\sigma_8$  when deriving final results. We detail our methods for sampling the multi-dimensional probability distribution functions of interest in Models 1 through 4 in Appendix C.

This approach to parameter fitting allows our estimates of the growth of structure and geometry to be independent of many model assumptions and can be used to put constraints on more general models of gravity and dark energy. However, they still rely on the

standard model in three ways. First, we assume that the processes in the early Universe that were responsible for setting up the linear matter power spectrum at recombination do not change significantly, which is true for the most popular models of modified gravity and dark energy. Second, we assume that growth at the level of linear perturbation theory is scale independent between the CMB epoch and the effective redshift of our sample. Third, we use GR to compute the perturbation theory corrections to the galaxy clustering predictions. The perturbation theory corrections are not large, and they are most important on small scales where  $\sigma_{\text{FoG}}^2$  also becomes important; therefore we cannot strongly constrain the amplitude of the higher-order corrections. Constraints on models with scale-dependent growth should be derived directly from the correlation function measurements and their covariance.

## 5.3 The meaning of $\sigma_8$

We follow the standard convention of denoting the amplitude of the matter power spectrum by  $\sigma_8^2$ , even though we restrict our analysis to scales  $s > 25 h^{-1} \text{Mpc}$ , so a different weighted integral over  $P(k)$  that is concentrated on larger scales would more accurately reflect our constraints on the growth rate of matter fluctuations. In particular, since  $P(k)$  is well-determined for  $k$  in  $\text{Mpc}^{-1}$  (e.g. White 2006), and the BAO scale provides a standard ruler with percent-level precision, our data constrain the amplitude of matter fluctuations on scales  $\gtrsim 36 \text{Mpc}$ . In practice, the tight constraints on the shape of  $P(k)$  means that differences arising from how one specifies its amplitude are small when computing parameter constraints, as long as  $h$  is well-determined in the model. In Model 2, we sample power spectra from the WMAP  $\Lambda$ CDM chain, and take the traditional value of  $\sigma_8$  to relate the model parameter  $f$  and the reported constraint  $f\sigma_8$ . In Model 4, we do not specify a value of  $h$  with each sampled power spectrum, so we normalize the power spectra by fixing  $\sigma_R$ , where  $R = 8/0.7 = 11.4 \text{Mpc}$ . For power spectra drawn from WMAP7  $\Lambda$ CDM chains,  $\sigma_R/\sigma_8 = 0.99 \pm 0.024$ ; the offset and variance between these parameters is negligible compared to our measurement errors on  $f\sigma_8$ .

## 6 RESULTS

In this section we present the results of fitting our analytic model for  $\xi_{0,2}(s)$  to the observed galaxy correlation functions. Figure 7 summarizes our constraints from the shape of the observed angle-averaged correlation function  $\xi_0(s)$ , while Figures 8 and 9 highlight our parameter constraints from the observed anisotropic galaxy clustering. Constraints on both the peculiar velocity amplitude and geometric quantities  $(1 + z_{\text{eff}})D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  are summarized in Table 1 for Models 2-4.

### 6.1 Goodness of fit and $\Lambda$ CDM results

We include 23 separation bins for both  $\ell = 0$  and  $\ell = 2$  in our  $\chi^2$  analyses. In this section, we consider models with increasing numbers of free parameters, and ask whether changes in  $\chi^2$  across the models indicate a preference for parameter values outside the predicted values from WMAP7 in the  $\Lambda$ CDM model. We first fix the underlying power spectrum to the one assumed for all of the mock catalogs, and also fix  $\sigma_{\text{FoG}}^2 = 21 \text{Mpc}^2$ , the best fit value to our  $N$ -body based mock galaxy catalogs. We vary the galaxy bias, and find a minimum  $\chi^2 = 45.7$  for 45 degrees of freedom, demonstrating that the mock galaxy catalogs used to validate our model and

Model	$b\sigma_8$	$f\sigma_8$	$D_V$ [Mpc]	$F$	$(1+z_{\text{eff}})D_A$ [Mpc]	$H$ [km s $^{-1}$ Mpc $^{-1}$ ]
2	$1.228^{+0.033}_{-0.032}$	$0.415^{+0.034}_{-0.033}$	-	-	-	-
3	$1.246^{+0.043}_{-0.046}$	-	$2076^{+42}_{-44}$	$0.683^{+0.026}_{-0.025}$	$2204 \pm 44$	$92.9^{+3.6}_{-3.3}$
4	$1.238^{+0.047}_{-0.050}$	$0.427^{+0.069}_{-0.063}$	$2070^{+43}_{-46}$	$0.675^{+0.042}_{-0.038}$	$2190 \pm 61$	$92.4^{+4.3}_{-4.0}$
WMAP7 $\Lambda$ CDM	-	$0.451 \pm 0.025$	$2009 \pm 42$	$0.6635^{+0.0084}_{-0.0073}$	$2113^{+53}_{-52}$	$94.2^{+1.4}_{-1.3}$

**Table 1.** The median and 68.3 per cent confidence level intervals on parameters  $b\sigma_8$ ,  $f\sigma_8$ , absolute distance scale  $D_V$  (Eqn. 15), Alcock-Paczynski parameter  $F$  (Eqn. 16), as well as derived parameters, comoving angular diameter distance ( $(1+z_{\text{eff}})D_A$ ) and expansion rate ( $H$ ). To obtain these constraints, we marginalize over  $\sigma_{\text{FoG}}^2$  and power spectrum shape parameters  $\vec{p}_s = \{\Omega_b h^2, \Omega_c h^2, n_s\}$  for Models 2-4, as described in Section 5.2. We interpret our measurements at the effective redshift of our galaxy sample,  $z_{\text{eff}} = 0.57$ .

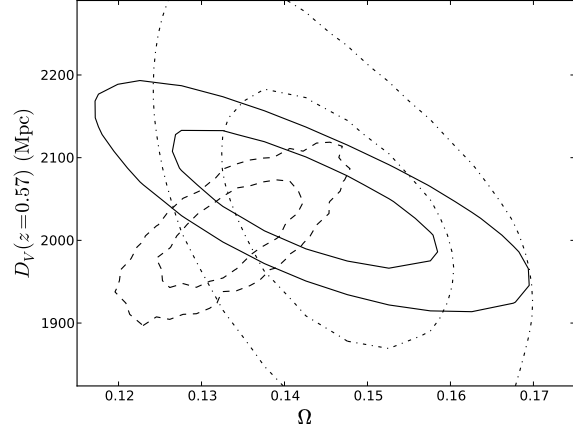
compute our covariance matrix are consistent with the observed galaxy clustering. If we allow  $\sigma_{\text{FoG}}^2$  to vary as well,  $\chi^2 = 42.1$  at  $\sigma_{\text{FoG}}^2 = 40 \text{ Mpc}^2$ . The difference indicates that we cannot expect a strong constraint on  $\sigma_{\text{FoG}}^2$  within our prior when other cosmological parameters are varying; it is important, however, to marginalize over this nuisance parameter, since it increases our uncertainty in  $f\sigma_8$ ; see the discussion in Section 6.5.

If we restrict ourselves to  $\Lambda$ CDM models consistent with WMAP7 (Model 1), we find a minimum  $\chi^2$  value of 39.3 at  $\Omega_m h^2 = 0.1395$  and  $H_0 = 68.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$  when  $b\sigma_8$  and  $\sigma_{\text{FoG}}^2$  are varied. Model 4 has the most free parameters: five describing the galaxy clustering and three specifying the linear matter power spectrum. In this case we find a minimum  $\chi^2$  of 39.0 for 41 degrees of freedom. This best-fitting model is shown with our measurements of the correlation function in Figures 3 and 4, and has parameter values  $b\sigma_8 = 1.235$ ,  $f\sigma_8 = 0.437$ ,  $\sigma_{\text{FoG}}^2 = 40 \text{ Mpc}^2$ ,  $D_A = 2184 \text{ Mpc}$ ,  $H = 91.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_m h^2 = 0.1364$ ,  $\Omega_b h^2 = 0.02271$ ,  $n_s = 0.967$ . We conclude that the observed  $\xi_{0.2}$  is fully consistent with the  $\Lambda$ CDM cosmology; changes in  $\chi^2$  values between the models do not indicate a significant preference for parameter values of  $f\sigma_8$ ,  $F$ , and  $D_V/r_s$  outside of the values predicted by WMAP7 in the  $\Lambda$ CDM model.

CMASS measurements also improve constraints compared to WMAP7 in the  $\Lambda$ CDM model:  $\Omega_m h^2 = 0.1363 \pm 0.0035$ ,  $\Omega_m = 0.283 \pm 0.017$ , and  $H_0 = (69.3 \pm 1.5) \text{ km s}^{-1} \text{ Mpc}^{-1}$ ; WMAP7 alone finds  $\Omega_m h^2 = 0.1334 \pm 0.0056$ ,  $\Omega_m = 0.266 \pm 0.029$ , and  $H_0 = (71.0 \pm 2.5) \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Comparison with the BAO-only results of Anderson et al. (2012) demonstrates that in this minimal model, nearly all of the additional information on these three parameters is coming from the BAO feature.

However, the shape of the measured galaxy correlation function does provide an independent probe of the underlying linear matter power spectrum. With a strong prior on  $\Omega_b h^2$  and  $n_s$  taken from the CMB, the clustering of galaxies is sensitive to the peak in the linear matter power spectrum, which depends on the horizon size at matter-radiation equality,  $\propto \Omega_m h^2$  at fixed effective number of relativistic species ( $N_{\text{eff}} = 3.04$  for the standard three neutrino species). The scale at which the peak appears depends on the low redshift distance relation, so the broadband shape of the angle-averaged galaxy power spectrum or correlation function constrains  $\Omega_m h^2 D_V(z_{\text{eff}})$ , in addition to the constraint on  $D_V(z_{\text{eff}})/r_s(z_{\text{drag}})$  that comes from the location of the BAO feature. We illustrate the constraining power of our dataset by fixing  $\Omega_b h^2 = 0.02258$  and  $n_s = 0.963$ , and computing the CMASS-only likelihood in the  $D_V - \Omega_m h^2$  plane shown in Figure 7 as the solid contours. For this exercise we use only the monopole ( $\ell = 0$ ) measurements, and find a minimum  $\chi^2$  value of 18.2 for 19 degrees of freedom (DOF).

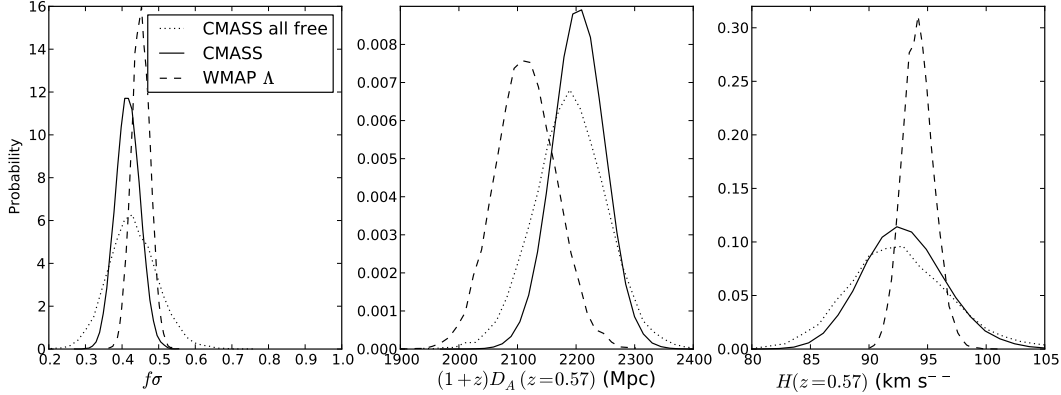
For comparison, we also isolate the broadband shape information by fitting to a no-wiggle power spectrum (Eisenstein & Hu 1998), which should primarily be sensitive to  $\Omega_m h^2 D_V(z_{\text{eff}})$ . Re-



**Figure 7.** Contours of  $\Delta\chi^2 = 2.30$  and 6.17 for fixed  $\Omega_b h^2 = 0.02258$  and  $n_s = 0.963$  for the monopole ( $\ell = 0$ ) galaxy clustering measurements alone (solid). For comparison, we also compute  $\chi^2$  using the “no-wiggles” power spectrum from Eisenstein & Hu (1998) (dash-dot) to isolate information from the broadband shape of the correlation function without the BAO feature; this fit is primarily sensitive to the apparent location of the peak in  $P(k)$ , which corresponds to the horizon size at matter-radiation equality,  $\propto \Omega_m h^2 D_V(z_{\text{eff}})$ . We also project the WMAP  $\Lambda$ CDM constraints onto these parameters, and show 68 and 95 per cent contours (dashed).

sults of this fit are shown as dash-dot contours in Figure 7. This model provides a poor fit to the measured correlation function, with  $\chi^2_{\text{min}} = 38.5$  for 19 DOF, indicating a strong preference for models with the expected BAO feature. However, we do find that the inferred value of  $\Omega_m h^2$  from the broadband shape of the measured galaxy correlation function is consistent with the prediction from the CMB.

Projecting the with-BAO model fits onto  $\Omega_m h^2$ , we find  $\Omega_m h^2 = 0.142 \pm 0.011$ . Translating the results of a similar analysis from Reid et al. (2010) for the SDSS-II LRG sample ( $z_{\text{eff}} = 0.31$ ) to the same assumptions yields  $\Omega_m h^2 = 0.141^{+0.010}_{-0.012}$ , while the WiggleZ analysis of emission line galaxies at  $z_{\text{eff}} \approx 0.6$  finds  $\Omega_m h^2 = 0.127 \pm 0.011$  (Blake, et al. 2011a). Strictly speaking, these constraints are not uncorrelated since they have a small amount of overlapping volume; neglecting their small correlation, the combined galaxy clustering estimate for  $\Omega_m h^2$  is  $0.137 \pm 0.0064$ , which marginalizes over the low redshift distance-redshift relation and is in excellent agreement with the WMAP7  $\Lambda$ CDM constraint of  $\Omega_m h^2 = 0.1334 \pm 0.0056$  (dashed contours in Figure 7).



**Figure 8.** One-dimensional constraints on  $f\sigma_8$ ,  $(1+z_{\text{eff}})D_A(z_{\text{eff}})$ , and  $H(z_{\text{eff}})$  under different model assumptions. The dashed curves indicate WMAP7-only  $\Lambda$ CDM. The solid (models 2 and 3) and dotted (model 4) curves are constraints derived from the CMASS  $\xi_{0,2}$  measurements with a WMAP7 prior on the underlying linear matter power spectrum  $P(k/\text{Mpc}^{-1})$ . The solid curves additionally use the  $\Lambda$ CDM parameters in the WMAP7 chains to fix either  $(1+z_{\text{eff}})D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  (left panel), or  $f\sigma_8$  (right two panels). All three constraints degrade when fitting for geometry and growth simultaneously using the CMASS observations.

## 6.2 Constraints on the peculiar velocity field amplitude

In Figures 8 and 9 we compare our constraints on the peculiar velocity field amplitude,  $f\sigma_8(z_{\text{eff}})$ , from Models 2 (solid) and 4 (dotted) to the predicted distribution from WMAP7 (dashed), assuming a flat  $\Lambda$ CDM cosmology. The 68 per cent confidence intervals for Models 2 and 4 are listed in Table 1. These measurements agree with the  $\Lambda$ CDM WMAP7 expectation,  $0.451 \pm 0.025$ .

## 6.3 Geometric constraints

Our tightest geometric constraint comes from the BAO feature in the monopole correlation function. The cosmological parameter dependence of the location of the BAO feature is given by the sound horizon at the drag epoch,  $r_s(z_{\text{drag}})$  (we use the definition in Eisenstein & Hu 1998). We find  $(D_V(z_{\text{eff}})/r_s(z_{\text{drag}}))/(D_V(z_{\text{eff}})/r_s(z_{\text{drag}}))_{\text{fiducial}} = 1.023 \pm 0.019$ . The difference between our best fit value and the pre-reconstruction fits to monopole correlation function presented in Anderson et al. (2012) and Sanchez et al. (2012) is due to our different choice of binning rather than our fitting methodology; we verified that with the same measurement values and covariance matrix, our method recovers the same central value as the result presented in Anderson et al. (2012).

Though our fits include information from the broadband shape of the correlation function, the resulting central value and error on  $D_V/r_s(z_{\text{drag}})$  are consistent with the fits performed in Anderson et al. (2012), which marginalize over the broadband shape of correlation function or power spectrum. We therefore conclude that essentially *all* of the information on the distance scale  $D_V$  is coming from the BAO feature in the correlation function, as was also true in the analysis of the SDSS-II LRG power spectrum (Reid et al. 2010). This can be seen in Fig. 7, where at fixed  $\Omega_m h^2$ , the constraint on  $D_V$  is 2.5 times weaker for the “no-wiggles” fit compared to the fit including the BAO feature. In addition, our central value and error on  $D_V/r_s$  are consistent when we fit  $\xi_0$  only, or  $\xi_0$  and  $\xi_2$  simultaneously.

Finally, we note that many of the small differences between the cosmological constraints presented here and those in our companion papers stem from slight differences in the best fitting value

for  $D_V/r_s(z_{\text{drag}})/(D_V(z_{\text{eff}})/r_s(z_{\text{drag}}))_{\text{fiducial}}$ . The correlation function and power spectrum post-reconstruction “consensus” value from Anderson et al. (2012) is  $1.033 \pm 0.017$ ; this value was used in cosmological parameter studies in that paper. Sanchez et al. (2012) found  $1.015 \pm 0.019$ , in agreement with the pre-reconstruction analysis of the correlation function presented in Anderson et al. (2012).

Once the WMAP7 prior on the underlying linear matter power spectrum  $p_s$  is included and information from the Alcock-Paczynski effect is included through  $\xi_2$ , the standard ruler from the CMB allows us to infer  $(1+z_{\text{eff}})D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  separately. Constraints from Model 3 (solid) and Model 4 (dotted) on  $(1+z_{\text{eff}})D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  are shown in Figures 8 and 9. Model 3 further uses the WMAP7  $\Lambda$ CDM prediction for  $f\sigma_8$  to disentangle the RSD and AP effects, and we find factors of 1.3 and 1.2 improvement in  $(1+z_{\text{eff}})D_A$  and  $H$  errors when adopting this additional assumption:  $(1+z_{\text{eff}})D_A = 2204 \pm 44$  ( $2190 \pm 61$ ) Mpc,  $H = 92.9^{+3.6}_{-3.3}$  ( $92.4^{+4.5}_{-4.0}$ )  $\text{km s}^{-1} \text{Mpc}^{-1}$ . In both models, the CMASS distance constraints are consistent with what is inferred from WMAP7 alone in a  $\Lambda$ CDM cosmology:  $(1+z_{\text{eff}})D_A(z_{\text{eff}}) = 2113^{+53}_{-52}$ ,  $H(z_{\text{eff}}) = 94.2^{+1.4}_{-1.3}$ . We compare the two-dimensional constraints on  $(1+z)D_A$  and  $H$  from Models 3 and 4 with the prediction from WMAP7 for a flat  $\Lambda$ CDM model in Fig. 9, which shows that CMASS constraints on  $(1+z)D_A$  and  $H$  are only weakly correlated.

## 6.4 Using our results

Our results may be used to test cosmological models which share the assumptions we have adopted in this analysis. Most importantly, we have assumed adiabatic and scale-invariant primordial fluctuations, and that the transfer function was computed assuming the standard number of massless neutrino species,  $N_{\text{eff}} = 3.04$ . We have assumed that the linear growth is scale-independent, and account for non-linear corrections using perturbation theory within general relativity. The code to evaluate our theoretical prediction as a function of the underlying linear matter power spectrum, **cosmoxi2d** is publicly available<sup>1</sup>. For most purposes, however, our results can

<sup>1</sup> <http://mwhite.berkeley.edu/CosmoXi2D>

be well-approximated by the following multivariate Gaussian likelihood for the parameters  $p_{3d} = \{f\sigma_8, F, (D_V/r_s)/(D_V/r_s)_{\text{fiducial}}\}$ , which should be interpreted at  $z_{\text{eff}} = 0.57$ :

$$\bar{p}_{3d} = \begin{pmatrix} 0.4298 \\ 0.6771 \\ 1.0227 \end{pmatrix} \quad (17)$$

$$10^3 \mathbf{C} = \begin{pmatrix} 4.509868 & 2.435891 & -0.01087251 \\ 2.435891 & 1.736087 & -0.06287155 \\ -0.01087251 & -0.06287155 & 0.3548373 \end{pmatrix} \quad (18)$$

Deviations from the Gaussian likelihood are significant only for points  $\gtrsim 3\sigma$  from the best fit values, where corrections to the likelihood surface of  $\xi_{0,2}$  will also become important.

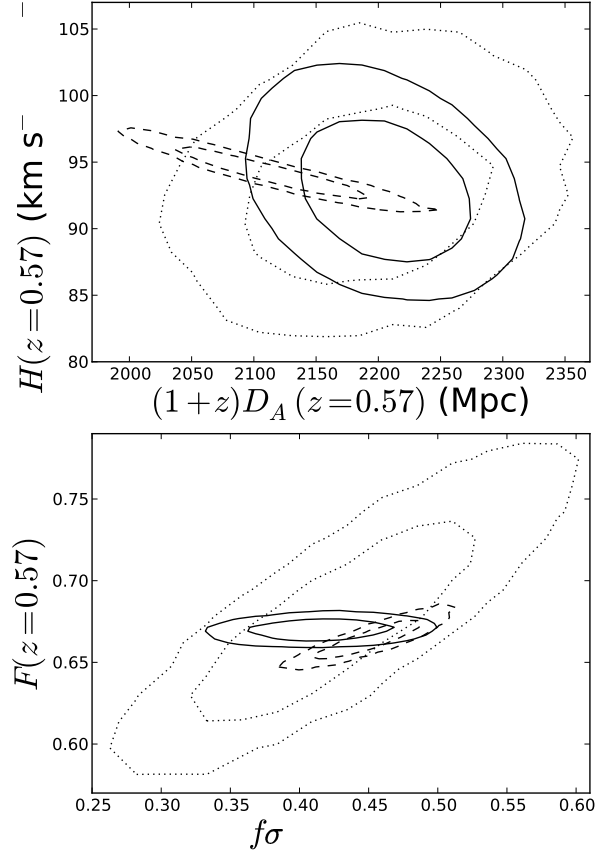
### 6.5 Key degeneracies and error budget

In this section we examine the main sources of uncertainty in our measurements of  $f\sigma_8(z_{\text{eff}})$  and  $F(z_{\text{eff}})$ . Both of these parameters affect the amplitude of the quadrupole, and so are partially degenerate; Figure 9 shows that our measurements are sufficiently sensitive to distinguish the two through their differing scale-dependence. We will consider in turn the uncertainty due to the nuisance parameter  $\sigma_{\text{FoG}}^2$  describing the intrahalo velocities of satellite galaxies (the “finger-of-god effect”), uncertainty in the underlying linear matter power spectrum, the redshift-distance relation (in the case of peculiar velocities), and the peculiar velocity field (in the case of the AP effect).

We first assess the impact of non-linearity in the covariance matrix on the error budget. Taking the number of galaxies in the present analysis, assuming  $\bar{n} = 3 \times 10^{-4} \text{ (h}^{-1} \text{ Mpc)}^{-3}$ , and using linear theory to evaluate the Fisher matrix (as in Reid & White 2011), we expect an uncertainty on  $f\sigma_8$  of  $\approx 0.021$  when only  $b\sigma_8$  and  $f\sigma_8$  are freely varied. Using our mock covariance matrix, we find an uncertainty of 0.029 on  $f\sigma_8$  at fixed  $\sigma_{\text{FoG}}^2 = 21 \text{ Mpc}^2$  and for  $P_{\text{lin}}(k)$ ,  $D_V$ , and  $F$  all fixed at their values in the mock catalog cosmology. This  $\sim 40$  per cent increase is primarily due to the non-linear/window function corrections to the covariance matrix highlighted in Figure 5. If we instead use the mock based covariance matrix to fit for both  $D_V$  and  $F$ , fixing  $\sigma_{\text{FoG}}^2$ ,  $P_{\text{lin}}(k)$ , and  $f\sigma_8$ , we find  $\sigma_F = 0.019$ . These results are summarized in Table 2.

#### 6.5.1 Degeneracy with $\sigma_{\text{FoG}}^2$

At fixed  $P(k)$  and geometric parameters, the Fisher matrix analysis indicates a factor of 2 increase in the  $f\sigma_8$  error, to 0.042, when  $\sigma_{\text{FoG}}^2$  is marginalized over without any prior, compared to when it is fixed at  $\sigma_{\text{FoG}}^2 = 21 \text{ Mpc}^2$ . The marginalized error on  $\sigma_{\text{FoG}}^2$  in the former case is  $14 \text{ Mpc}^2$ . Therefore, the hard prior  $0 < \sigma_{\text{FoG}}^2 < 40 \text{ Mpc}^2$  substantially reduces this source of uncertainty. Table 2 indicates an increase of only 15 per cent in the error on  $f\sigma_8$ , to 0.033, when we marginalize over  $\sigma_{\text{FoG}}^2$  within our hard prior. Similarly, marginalizing over  $\sigma_{\text{FoG}}^2$  increases the error on  $F$  from 0.019 to 0.022. Therefore, further reduction in the uncertainty on  $\sigma_{\text{FoG}}^2$  with more detailed modeling of the small-scale clustering would only allow a slight reduction in the errors. However, since our fits indicate a slight preference for  $\sigma_{\text{FoG}}^2 = 40 \text{ Mpc}^2$  compared to the fiducial  $21 \text{ Mpc}^2$ , in future work we will revisit our choice of prior after a re-analysis of small-scale CMASS clustering with a larger data set.



**Figure 9.** Upper panel: 68 and 95 per cent confidence regions for the comoving angular diameter distance and expansion rate at  $z = 0.57$  from CMASS anisotropic clustering constraints when  $f\sigma_8$  is varied over the WMAP7+GR flat  $\Lambda$ CDM prior (Model 3; solid) and when  $f\sigma_8$  is simultaneously fit (Model 4; dotted). Lower panel: 68 and 95 per cent confidence regions for  $f\sigma_8(z = 0.57)$  and Alcock-Paczynski parameter  $F(z = 0.57)$  inferred from CMASS anisotropic clustering (Model 4; dotted). These two parameters are partially degenerate, and their differing scale-dependence allows us to constrain each separately. The solid contour shows the constraint when a WMAP7+GR flat  $\Lambda$ CDM prior is used on  $F$ . In both panels we show for comparison the predictions from WMAP7 when a standard GR, flat  $\Lambda$ CDM cosmology is assumed (dashed).

#### 6.5.2 Uncertainty in the underlying linear matter $P(k)$

Of the three parameters defining the shape of the underlying linear matter  $P(k)$  in our analysis, the uncertainty in  $\omega_c = \Omega_c h^2$  is the largest, particularly in the context of the constraining power of the CMASS measurements. For this comparison, we therefore hold  $\Omega_b h^2 = 0.02258$  and  $n_s = 0.963$  fixed, and examine how the central values and uncertainties in  $b\sigma_8$ ,  $f\sigma_8$  or  $F$  change as a function of  $\omega_c$ , with all other parameters held fixed. Empirically, the uncertainties on these parameters do not depend much on  $\omega_c$ . Therefore, we can estimate the impact on the uncertainty in  $\omega_c$  through the dependence of the central value of parameter  $p$  on  $\omega_c$ , in units of the uncertainty on those quantities:

$$s = \frac{\Delta p / \sigma_p}{\Delta \omega_c / \sigma_{\omega_c}}, \quad (19)$$

Marginalized parameters	$\sigma_{b\sigma_8}$	$\sigma_{f\sigma_8}$	$\sigma_F$	Model
Fisher	0.019	0.021	-	-
Fisher, $\sigma_{\text{FoG}}^2$	0.024	0.042	-	-
-	0.023	0.029	-	-
$\sigma_{\text{FoG}}^2$	0.025	0.033	-	-
$\sigma_{\text{FoG}}^2, P(k)$	0.033	0.033	-	-
$\sigma_{\text{FoG}}^2, P(k), \text{geometry}$	0.033	0.034	-	2
-	0.037	-	0.019	-
$\sigma_{\text{FoG}}^2$	0.038	-	0.022	-
$\sigma_{\text{FoG}}^2, P(k)$	0.046	-	0.022	-
$\sigma_{\text{FoG}}^2, P(k), \text{growth}$	0.046	-	0.026	3
$\sigma_{\text{FoG}}^2$	0.047	0.069	0.042	-
$\sigma_{\text{FoG}}^2, P(k)$	0.050	0.069	0.042	4

**Table 2.** We examine how uncertainty in various quantities entering our analysis impacts the 68 per cent confidence level intervals on parameters  $b\sigma_8$ ,  $f\sigma_8$ , and  $F$ . For comparison, the first two rows show the predictions from a simple linear theory Fisher matrix analysis (as in Reid & White 2011) with  $\bar{n} = 3 \times 10^{-4} (h^{-1}\text{Mpc})^{-3}$ , the number of galaxies in the present analysis, and ignoring all window function effects. When  $\sigma_{\text{FoG}}^2$  is marginalized over for the measurements (but *not* for the Fisher matrix analysis), we maintain a hard prior  $0 < \sigma_{\text{FoG}}^2 < 40 \text{ Mpc}^2$ . Uncertainty in the underlying linear  $P(k)$  is derived from WMAP7 data, under the assumptions of Gaussian, adiabatic, power law initial conditions with  $N_{\text{eff}} = 3.04$  massless neutrino species. Uncertainty in geometry [ $D_V(z_{\text{eff}})$  and  $F(z_{\text{eff}})$ ] or growth [ $f\sigma_8$ ] is taken to be the uncertainty on these quantities derived from WMAP7 in a  $\Lambda\text{CDM}$  cosmology. The last column highlights the cases corresponding to Models 2 through 4.

where  $\sigma_p$  is the uncertainty at fixed  $\omega_c$ . For Gaussian probability distribution functions in  $P(p|\omega_c)$  and  $P(\omega_c)$ , the uncertainty on  $p$  when marginalized over  $\omega_c$  is increased by  $\sqrt{1+s^2}$ . We find that  $s \lesssim 0.1$  for  $f\sigma_8$  and  $s = 0.13$  for  $F$ . Therefore, the current uncertainty in the underlying linear matter  $P(k)$  is negligible for the purpose of deriving constraints from the anisotropic clustering of CMASS galaxies. This justifies the use of a fixed power spectrum shape in the WiggleZ analyses of anisotropic clustering (Blake, et al. 2011b,c). However, the uncertainty in  $P(k)$  does increase the uncertainty in  $b\sigma_8$ , so applications such as galaxy-galaxy lensing/galaxy clustering combinations (e.g., Reyes et al. 2010) should marginalize over this additional uncertainty.

### 6.5.3 Uncertainty in $\Lambda\text{CDM}$ geometry

Table 2 indicates that marginalizing over the uncertainty in the geometric quantities  $(1+z_{\text{eff}})D_A(z_{\text{eff}})$  and  $H(z_{\text{eff}})$  [or equivalently,  $D_V(z_{\text{eff}})$  and  $F(z_{\text{eff}})$ ] contributes negligibly to the uncertainty in  $f\sigma_8$  for WMAP7 uncertainties when a  $\Lambda\text{CDM}$  redshift-distance relation is assumed.

### 6.5.4 Uncertainty in $\Lambda\text{CDM}$ growth of structure

In the  $\Lambda\text{CDM}$  model, WMAP7 constrains  $\Omega_m$  to  $\sim 11$  per cent; this translates into a relatively large uncertainty in the predicted growth rate of structure ( $f\sigma_8$ ) of 5.5 per cent. Marginalizing over this uncertainty increases our uncertainty on  $F$  from 0.022 to 0.026. Weak lensing (e.g., Munshi et al. 2008) and/or cluster abundances (e.g., Vikhlinin et al. 2009; Rozo et al. 2010) reduce the  $\Lambda\text{CDM}$  uncertainty on a combination of  $\sigma_8$  and  $\Omega_m$  similar to the GR prediction for peculiar velocities,  $\sigma_8\Omega_m^{0.55}$ , and thus could potentially be used to reduce the uncertainty on  $f\sigma_8$  in the  $\Lambda\text{CDM}$  growth scenario.

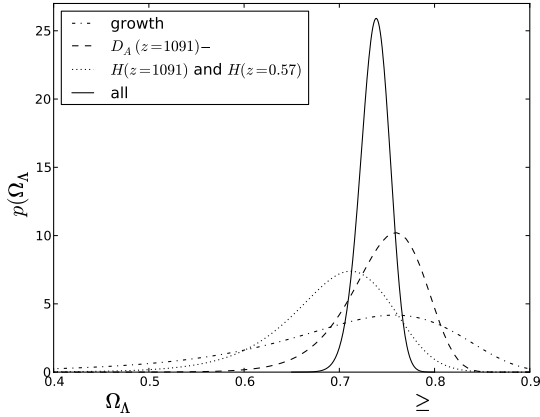
In the present work, we do not explore other dataset combinations besides WMAP7 and CMASS.

## 6.6 Comparison with previous measurements at $z \approx 0.6$

The WiggleZ survey has recently analyzed the anisotropic clustering of bright emission line galaxies over a broad redshift range (Blake, et al. 2011b,c). Their growth rate constraints assumed a fixed underlying linear matter power spectrum and redshift-distance relation, for which our error is 0.033. With a factor  $\sim 4$  fewer galaxies, they achieve comparable precision and good agreement with our central value in their  $z = 0.6$  bin:  $f\sigma_8(z = 0.6) = 0.43 \pm 0.04$ . In their analysis they include modes up to  $k_{\text{max}} = 0.3 h \text{ Mpc}^{-1}$  and marginalize over a Lorentzian model to account for small scale nonlinearities. We find that the quoted WiggleZ error is in good agreement with our Fisher matrix prediction if we assume  $\bar{n} = 2.4 \times 10^{-4} (h^{-1}\text{Mpc})^{-3}$ ,  $b = 1.1$ ,  $N_{\text{gal}} = 60227$ , a Gaussian damping  $\sigma_{\text{Gauss}} = 300 \text{ km s}^{-1}$  and  $s_{\text{min}} = 1.15\pi k_{\text{max}}^{-1} = 12 h^{-1}\text{Mpc}$ , i.e., a factor of two smaller scale than we have adopted for our analysis. Exploration using the Fisher matrix suggests that the difference in number densities between CMASS and WiggleZ has a negligible impact on the uncertainties, while the lower bias of their sample implies a 15 per cent (10 per cent) improvement at  $s_{\text{min}} = 25 (12) h^{-1} \text{ Mpc}$ . By far the dominant difference arises because they are fitting out to  $k_{\text{max}} = 0.3 h \text{ Mpc}^{-1}$ ; their error would increase by a factor of 2 if they adopted our minimum scale. Another issue is that our mock covariance matrix, which accounts for both nonlinear growth of structure and our complex survey geometry, yield errors 40 per cent larger than our naive Fisher matrix analysis would predict, with the difference between the linear and mock covariance matrices increasing on small scales (for  $\xi_2$ ). While it is not clear how this difference scales with galaxy bias or depends on survey geometry, the WiggleZ use of linear theory covariance matrices down to much smaller scales could cause their uncertainty to be underestimated.

Another recent WiggleZ analysis (Blake, et al. 2011c) simultaneously fits  $F = 0.68 \pm 0.06$  and  $f\sigma_8 = 0.37 \pm 0.08$  at  $z = 0.6$  for a fixed underlying  $P(k)$ , and makes use of modes with  $k < 0.2 h \text{ Mpc}^{-1}$ , which can be compared with the last two lines of Table 2 and Model 4 in Table 1. As in the previous discussion, our best fit values for these parameters are consistent, and using clustering information on small scales (i.e., larger  $k_{\text{max}}$ ) permits a factor of  $\sim 1.4$  tighter constraint on  $F$  at a fixed number of galaxy spectra compared with our clustering analysis of CMASS galaxies. However, the higher bias of CMASS galaxies permits relatively tighter constraints on  $F$  than  $f\sigma_8$  as compared with WiggleZ.

The WiggleZ and BOSS CMASS surveys target very different galaxy types, which will have different nonlinear properties and modeling uncertainties. Without a detailed study it is impossible to understand the robustness of various assumptions for the nonlinear distortions, particularly when multiple sources of nonlinearity contribute. These can potentially cancel when examining clustering, meaning that goodness-of-fit must be used carefully when assessing robustness and potential biases in a model fit. In this work we have adopted a conservative approach based on the best model available that has been compared with a large volume of  $N$ -body mock galaxy catalogs, and only fit over those scales where we are confident that the signal is dominated by the quasilinear velocity field of interest, and where the impact of small-scale random motions can be simply modeled and marginalized over.



**Figure 10.** Joint constraints on  $\Omega_\Lambda$  from the combination of CMB distance and normalization priors and our measurements of  $D_A$ ,  $H$ , and  $f\sigma_8$ . These constraints are independent of the growth and expansion history at redshifts lower than our galaxy sample, but assume a flat  $\Lambda$ CDM cosmology between the CMASS sample and the CMB, as well as the other model assumptions detailed in Section 5.1.

## 7 DISCUSSION

We have analyzed the anisotropic clustering of BOSS DR9 CMASS galaxies with an accurate analytic model for the monopole and quadrupole correlation functions, which we have validated using a large volume of  $N$ -body based mock galaxy catalogs. The combination of the BAO standard ruler and the Alcock-Paczynski effect allows us to separately constrain the comoving angular diameter distance and Hubble expansion rate at the effective redshift of our sample,  $z = 0.57$ , while redshift space distortions allow us to constrain the amplitude of the peculiar velocity field, a direct measurement of the growth rate of structure,  $d\sigma_8/d\ln a$ . Table 1 summarizes our constraints under several assumptions. Figure 9 shows the degeneracy between the Alcock-Paczynski parameter  $F$  and the growth rate of structure in our measurements; this explains why our constraints improve considerably if we make further model assumptions about the geometry or growth. In the most general case where all three parameters vary independently, we find  $d\sigma_8/d\ln a = 0.43^{+0.069}_{-0.063}$ ,  $(1 + z_{\text{eff}})D_A(z_{\text{eff}}) = 2190 \pm 61$  Mpc and  $H(z_{\text{eff}}) = 92.4^{+4.5}_{-4.0}$  km s $^{-1}$  Mpc $^{-1}$ .

To illustrate the cosmological constraining power of our measurements, we summarize our results as three distinct tests of a minimal  $\Lambda$ CDM cosmology (see Section 5.1) that link the observed CMB anisotropies at  $z \approx 1091$  and CMASS galaxy fluctuations at  $z \approx 0.57$ , independent of the expansion history and growth of structure in the universe at redshifts below our sample ( $z \lesssim 0.57$ ). We use a flat  $\Lambda$ CDM model to illustrate our constraining power on the behavior of the universe at  $z > 0.57$ , and explore more general cosmological models in our companion paper, Samushia et al. (2012).

*Is the shape of the power spectrum of matter density fluctuations inferred from the CMB consistent with the one inferred from galaxy fluctuations after a factor of  $\sim 4 \times 10^5$  amplification?*

If the dominant component of the energy density is “cold”, then in the linear regime perturbation growth in General Relativity is scale-independent. Modulo our corrections for non-linear

$\Lambda$ CDM evolution and galaxy biasing ( $\sim 10$  per cent on the scales we analyse), the shapes of the linear matter power spectrum inferred from the CMB and galaxy clustering are consistent: our best fit  $\Lambda$ CDM model gives  $\chi^2 = 39.3$  for 44 degrees of freedom. We quantify this statement further using our fit to the location of the broad turnover in  $P(k)$ , which indicates the horizon size at matter-radiation equality and thus the physical matter density, assuming the radiation density is known. Combining our constraint ( $\Omega_m h^2 = 0.142 \pm 0.011$ ) with those in the literature for the SDSS-II LRG sample and WiggleZ yields  $\Omega_m h^2 = 0.137 \pm 0.0064$ , where the error neglects the expected small but non-zero covariance between the galaxy samples, and also fixes  $\Omega_b h^2$  and  $n_s$  to best fit CMB values. The CMB constraint is only slightly more precise than the combined galaxy measurement ( $\Omega_m h^2 = 0.1334 \pm 0.0056$ ), and the two are in excellent agreement. With this test passed, in the rest of our analysis we make use of the full matter power spectrum (rather than just the BAO feature) as a standard ruler in galaxy clustering measurements. See Sanchez et al. (2012) for an exploration of other cosmological models using the CMASS monopole correlation function shape. In particular, the good agreement between constraints on the linear matter power spectrum from the CMB and CMASS galaxy clustering limit the allowed contribution from species such as massive neutrinos which induce scale-dependent growth.

*Do our constraints on the geometry of the universe require dark energy at  $z \gtrsim 0.57$ ?*

Distance constraints from both the CMB and the BAO feature are determined relative to the sound horizon at  $z_\star \approx 1091$ , the redshift of decoupling, and at  $z_{\text{drag}} \approx 1020$ , the redshift when baryons were released from the Compton drag of the photons<sup>2</sup>. Within the constraints of our cosmological model assumptions (see Section 5.1),  $r_s(z_\star)$  and  $r_s(z_{\text{drag}})$  depend only on  $\Omega_m h^2$  and  $\Omega_b h^2$ . We project the observed *difference* in comoving angular diameter distance  $\Delta D_A = (1 + 1091)D_A(z = 1091) - (1 + 0.57)D_A(z = 0.57)$ ,

$$\Delta D_A = \int_{0.57}^{1091} \frac{dz}{H(z)} \quad (20)$$

onto the parameter  $\Omega_\Lambda$  in a flat  $\Lambda$ CDM cosmology and marginalize over the other parameters. The result is shown by the dashed line in Figure 10:  $\Omega_\Lambda = 0.76 \pm 0.04$ . We quote the maximum likelihood and 68 per cent confidence region around it unless otherwise noted. We have purposely chosen a variable that is independent of the expansion history at  $z < 0.57$ . Similarly, the CMB constraint on  $\Omega_m h^2$  with our measurement of  $H(z = 0.57)$  requires  $\Omega_\Lambda = 0.71^{+0.06}_{-0.05}$  between  $z = 0.57$  and  $z = 1091$  (dotted curve in Figure 10).

*Does the observed growth rate of structure at  $z = 0.57$  require dark energy?*

Because WMAP places such a tight constraint on the amplitude of curvature perturbations deep in the matter-dominated epoch at  $k = 0.027$  Mpc $^{-1}$  (1.8 per cent), we can translate our measurement of  $f\sigma_8$  into a constraint on  $dD/d\ln a$ , where  $D$  is the usual growth function that depends only on  $\Omega_m$  or  $\Omega_\Lambda$  in a flat  $\Lambda$ CDM

<sup>2</sup> We follow the WMAP7 analysis and adopt the fitting formulae for  $z_\star$  and  $z_{\text{drag}}$  from Hu & Sugiyama (1996) and Eisenstein & Hu (1998), respectively.

cosmological model:

$$D(z) = \frac{5\Omega_m}{2} \frac{H(z)}{H_0} \int^{1/(1+z)} \frac{da H_0^3}{[aH(a)]^3}. \quad (21)$$

To do so, we require the additional cosmological model assumptions listed in Section 5.1. We marginalize over the WMAP7 uncertainties in the parameters that convert curvature perturbations to the integrated amplitude of matter perturbations on scales of  $R = 8/0.7 = 11.4$  Mpc,  $\sigma_R$ , namely  $\Omega_m h^2$ ,  $\Omega_b h^2$ , and  $n_s$ . Section 5.3 details the relation between  $\sigma_R$  and  $\sigma_8$ , which can be regarded as equal for our purposes. This comparison between the fluctuation amplitude at  $z = 1091$  and  $z = 0.57$  requires  $\Omega_\Lambda$  within  $[0.59, 0.81]$  (central 68 per cent confidence), with a maximum likelihood at 0.76. The distribution is shown as the dot-dashed curve in Figure 10.

In combination with the CMB, both our geometric and growth rate constraints require a value of  $\Omega_\Lambda$  at  $0.57 < z < 1091$  that is consistent with the concordance model expectation. Combining all three constraints with the WMAP7 distance and normalization priors, and marginalizing over  $\Omega_b h^2$  and  $\Omega_m h^2$ , we find  $\Omega_\Lambda = 0.74^{+0.016}_{-0.015}$  (solid curve in Figure 10).

Our analysis required further assumptions compared with a BAO-only analysis (Anderson et al. 2012) that constrains  $D_V(z_{\text{eff}})/r_s(z_{\text{drag}})$ . However, we were able to perform tests of the scale-independence and growth rate of cosmic structure between recombination and  $z \approx 0.57$ , as well as to break the degeneracy between  $(1 + z_{\text{eff}})D_A$  and  $H$  using the Alcock-Paczynski test. So far, our measurements do not unveil any deviations from the minimal  $\Lambda$ CDM model we have examined. Under the assumption of a  $\Lambda$ CDM cosmology extending to  $z = 0$ , the BAO feature adds the most constraining power to WMAP7 on  $\Lambda$ CDM parameters; we find  $\Omega_m h^2 = 0.136 \pm 0.0035$ ,  $\Omega_m = 0.283 \pm 0.017$ , and  $H_0 = 69.3 \pm 1.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$  in this model. We anticipate statistical improvements on these results with the completed BOSS galaxy dataset covering a footprint three times larger, as well as developments in the theoretical modeling that will allow tighter cosmological constraints.

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## APPENDIX A: OBSERVATIONAL UNCERTAINTIES

### A1 Uncertainties in radial distribution

To compute overdensities of the galaxy field, defined as

$$\delta(z, \hat{\Omega}) = \frac{\rho_0(z, \hat{\Omega}) - \rho(z, \hat{\Omega})}{\rho_0(z, \hat{\Omega})}, \quad (A1)$$

we must know the unperturbed density field  $\rho_0(z, \hat{\Omega})$ . While the angular selection function of the survey is usually well known, the radial distribution is not easy to model accurately. Usually, the unperturbed radial distribution of galaxies is modeled from the distribution of observed redshifts by either shuffling them or splining with a smooth curve. Different ways of constructing a random catalog will result in different estimates of correlation function (see, e.g., Samushia, Percival, & Raccanelli 2011; Ross et al. 2012).

To estimate the magnitude of this effect on the measurements of the moments of correlation function we take 600 mock catalogs of CMASS sample and apply simplified version of our analysis to the measurements produced using different ways of reconstructing  $n(z)$ . We reconstruct  $n(z)$  first by shuffling “observed” redshifts in mock catalogs and then by splining that distribution with 10, 20 and 30 node cubic spline fits.

We find that the ways of reconstructing  $n(z)$  for random catalogs do not affect the RSD measurements significantly. In our current analysis we are using the “shuffled” catalogs since they introduce the least bias in the measurements of  $\xi_\ell(r)$  (Ross et al. 2012).

### A2 Effects of Close Pairs, redshift errors and redshift failures

We are unable to obtain redshifts for  $\sim 5$  per cent of the galaxies due to fiber collisions – no two fibers on any given observation can be placed closer than  $62''$ . At  $z \approx 0.5$  this  $62''$  exclusion corresponds to  $0.4 h^{-1} \text{ Mpc}$ . Redshifts for some of the collided galaxies can be reclaimed in regions where plates overlap, but the remaining exclusion must be accounted for. We account for fiber-collided galaxies by assigning its weight to its nearest neighbor on the sky. Tests on mock catalogs presented in (Guo, Zehavi, & Zheng 2011) indicate that the nearest neighbor correction adopted in this work is accurate to better than 1 per cent for both  $\xi_0$  and  $\xi_2$  at the scales used in our analysis.

Redshift failures are discussed in detail in Ross et al. (2012); we also correct for them with a nearest-neighbor upweighting



scheme. Redshift measurement errors smooth the apparent galaxy density field, in the same fashion as described with our nuisance parameter  $\sigma_{\text{FoG}}^2$ . The median redshift error for our sample is 42 km s<sup>-1</sup>, which translates into an additive contribution to  $\sigma_{\text{FoG}}^2 < 1 \text{ Mpc}^2$ .

## APPENDIX B: ACCURACY AND IMPLEMENTATION OF THE THEORETICAL MODEL

### B1 Wide Angle Effects

Equations 10 and 12 assume that a “plane-parallel” approximation, which states that the sample is far enough from the observer so that all line-of-sights are parallel to each other, is accurate enough. This approximation will fail at some scale for wide surveys (see e.g. Papai & Szapudi 2008). We approximate the scale dependent magnitude of wide-angle effects for our sample in a similar manner to Samushia, Percival, & Raccanelli (2011). The effective redshift of our galaxy sample is  $z = 0.57$ , which in the best-fit WMAP7 cosmology corresponds to a comoving distance of approximately 1500  $h^{-1}\text{Mpc}$ . The largest scale that we consider in this analysis is 160  $h^{-1}\text{Mpc}$ , which corresponds to an opening angle of about 3 degrees. We estimate the wide-angle corrections to be at most 10 per cent of the statistical errors on the largest scales and do not try to correct for this effect in current analysis.

### B2 *cosmoxi2d* code implementation

Reid & White (2011) demonstrated that Eqn. 12 provides an accurate description of  $\xi_g^s(r_\sigma, r_\pi)$  when real space clustering and velocity statistics inside the integrand are measured directly from  $N$ -body simulations. Moreover, in the regime where  $b \approx 2$  (appropriate for CMASS galaxies), the real space clustering and velocity statistics can be computed analytically with sufficient precision to predict  $\xi_g^s(r_\sigma, r_\pi)$ . In detail, this “sweet spot” in the precision of the model arises because the functions  $v_{12}(r)$  and  $\sigma_{12,\parallel/\perp}^2(r)$  entering Eqn. 12 were evaluated as a function of cosmological parameters in standard perturbation theory accounting for only the linear bias of the tracer. At the redshift of interest, the second order bias  $b_{2L}$  crosses zero near  $1 + b_{1L} = 2$ , and the calculation of these functions neglecting  $b_{2L}$  is sufficient for our purposes. We therefore caution against the use of our code for tracers with bias substantially different from 2. Note that  $\xi_g^s(r)$  is evaluated in LPT (Matsubara 2008b) and includes the contribution to the real space clustering from second-order Lagrangian bias<sup>3</sup>.

Standard perturbation theory is known to have inaccuracies in describing the BAO feature, and we found that on BAO scales errors in our perturbation theory calculation of pairwise velocity statistics caused inaccuracies in the prediction for  $\xi_g^s(r_\sigma, r_\pi)$ . However, on scales  $s \gtrsim 70 \text{ } h^{-1} \text{ Mpc}$ , the redshift space version of LPT (Matsubara 2008b) is very accurate (see figure 2 of Reid & White 2011). We therefore interpolate between our evaluation of Eqn. 12 at smaller scales and LPT on large scales, with the transition fixed at 100 Mpc.

Finally, we model the effect of galaxy intrahalo velocities (traditional “fingers-of-god”) by convolving our model  $\xi(r_p, r_\pi)$  with

an additional Gaussian velocity dispersion of variance  $\sigma_{\text{FoG}}^2$ . Using the Gaussian form allows  $\sigma_{\text{FoG}}^2$  to be included directly in the Gaussian in Eqn. 12 for faster evaluation of our model as we explore cosmological parameter space.

We have developed a code to numerically evaluate Eqn. 12 as well as all the relevant perturbation theory integrals as a function of an input linear matter power spectrum and nuisance parameters  $b$  and  $\sigma_{\text{FoG}}^2$ . The Alcock-Paczynski effect is easily incorporated using Eqn. 13 before computing Legendre polynomial moments from  $\xi_g^s(r_\sigma, r_\pi)$ . The internal units of *cosmoxi2d* are Mpc, in which the underlying linear matter power spectrum is most tightly constrained. The code is publicly available<sup>4</sup>.

### B3 Model Accuracy

Because we have such a large volume of simulations, we can use the difference between the theoretical model at the known cosmological parameters of the  $N$ -body simulation and the measured correlation function from the mock galaxy catalogs to quantify our theoretical systematic error; we find  $\Delta\chi^2 = 0.29$  at the best fit value of  $\sigma_{\text{FoG}}^2$ . We also compare the scale dependence of the model error with the 5 parameters we are fitting, and find  $< 0.25\sigma$  shifts compared to the *unmarginalized* uncertainties on all parameters (i.e., the uncertainty on each parameter if all the others were known perfectly). Therefore we conclude that our systematic error is negligible in the context of this analysis.

Another concern is that the model becomes inaccurate rapidly on scales smaller than our minimum fitting scale,  $s_{\text{min}}$ . When Alcock-Paczynski parameters vary, scales smaller than  $s_{\text{min}}$  contribute to the model. However, we have verified that for variations of 10 per cent in  $D_V$  and/or 20 per cent in  $F$  (i.e., much larger than the final uncertainties), the theoretical error induces  $< 0.5\sigma$  shifts in all parameters compared with the unmarginalized uncertainties.

Finally, we point out that unlike in linear theory, our model depends on  $b$ ,  $f$ , and  $\sigma_8$  separately, rather than the only the combinations  $b\sigma_8$  and  $f\sigma_8$ . However, at the particular  $b\sigma_8$  and  $f\sigma_8$  values of our sample, we find that changes in the predicted  $\xi_{0.2}$  with  $\sigma_8$  can be absorbed by changes in the value of  $b\sigma_8$ . Quantitatively, for a  $\pm 10$  per cent change in away from our fiducial  $\sigma_8(z_{\text{eff}}) = 0.61$  and at fixed  $P(k)$ , AP parameters, and  $\sigma_{\text{FoG}}^2 = 21 \text{ Mpc}^2$ ,  $b\sigma_8$  shifts by  $\approx \pm 1.3\sigma$  with no measurable shift in the central value of  $f\sigma_8$ .

### B4 Propagating uncertainties in the galaxy-halo mapping to

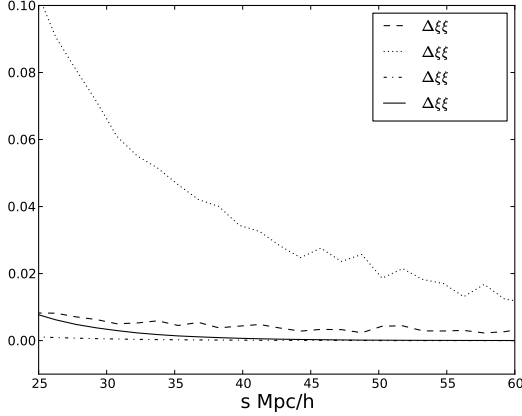
$$\sigma_{\text{FoG}}^2$$

As shown in Figure 6, our model with  $\sigma_{\text{FoG}}^2 = 21 \text{ Mpc}^2$  fits the  $\xi_{0.2}(s)$  of our mock galaxy catalogs, based on the best-fitting HOD in White et al. (2011). In this section we quantify how uncertainties in both the theoretical modeling and data analysis cause uncertainty in the expected value of  $\sigma_{\text{FoG}}^2$  for the CMASS sample. We address several aspects of this problem separately.

- *One-halo vs two-halo contributions to  $\xi_{0.2}$  and fiber collision corrections:* The formalism of the halo model distinguishes between “one-halo” and “two-halo” pairs depending on whether the two galaxies occupy the same or different halos. In Figure B1 we show the total change in  $\xi_{0.2}$  due to satellite galaxy intrahalo velocities (IHV) by the dashed (dotted) curves in our mock catalogs. The dash-dot and solid curves show the contributions to this change

<sup>3</sup> We relate the second order Lagrangian bias,  $b_{2L}$ , to the first order bias,  $b_{1L}$ , through the peak background split; these parameters are not varied independently.

<sup>4</sup> <http://mwhite.berkeley.edu/CosmoXi2D>



**Figure B1.** The dashed (dotted) lines show the fractional change in  $\xi_0$  ( $\xi_2$ ) in our mock galaxy catalogs due to satellite galaxy intrahalo velocities (IHV; also shown in Figure 6). We isolate the contribution of pairs of galaxies occupying the same halo in our mock catalogs (i.e., “fingers-of-god”), shown as the dot-dashed (solid) curves for  $\xi_0$  ( $\xi_2$ ). On the scales of interest, the dominant effect of IHV is a net diffusion of pairs from small scales (where  $\xi$  is larger) to larger scales.

in  $\xi_{0,2}$  from one-halo pairs, which are localized to along the LOS with  $r_\sigma \lesssim 1 h^{-1}\text{Mpc}$ . Because the one-halo pairs contribute to such a small  $d\mu$ , they can be neglected on the scales used in our cosmological parameter fits,  $s \gtrsim 25 h^{-1}\text{Mpc}$ . This fact is important to establish since our method for fiber collision corrections will correctly recover the distribution of pair separations for pairs of galaxies with separations larger than the fiber collision scale, but suppresses the contribution of pairs of galaxies at the fiber collision scale ( $r_\sigma \lesssim 0.5 h^{-1}\text{Mpc}$ ).

- *Uncertainty in the HOD at fixed cosmology:* Uncertainties in the HOD parameters will introduce an uncertainty in  $\sigma_{\text{FoG}}^2$ . While  $\sigma_{\text{FoG}}^2$  is roughly proportional to the satellite fraction, it also depends on the distribution of host halo mass – increasing  $\alpha$  and  $\kappa$  increases host halo mass at fixed satellite fraction, which increases  $\sigma_{\text{FoG}}^2$ . We use MCMC chains from White et al. (2011) to estimate our uncertainty on  $\sigma_{\text{FoG}}^2$  at fixed cosmology to be  $6 \text{ Mpc}^2$ .

- *Breakdown of the “central” galaxy assumption:* The analysis of Skibba et al. (2011) suggests that the brightest galaxy in a halo is not always the “central” one. We test the impact of relaxing our assumption that the velocity of mock central galaxies have no intrahalo velocities by assigning them the intrahalo velocity of a random dark matter particle halo member in our simulations some fraction  $p$  of the time. When we have more than one galaxy in a halo, we assume that the chance of not including the “central” galaxy in our sample is lower,  $\propto p^{N_{\text{gal}}}$ . For  $p = 0.3$ , we find  $\sigma_{\text{FoG}}^2$  at our fiducial HOD is increased by  $9 \text{ Mpc}^2$ .

- *Variations in the halo mass function with cosmological parameters:* A broad range of observations shows good agreement between the concordance  $\Lambda\text{CDM}$  halo mass function and the multiplicity of galaxy groups and clusters (e.g., Rozo et al. 2010; Allen, Evrard, & Mantz 2011), so this uncertainty is subdominant: even if our fiducial HOD masses were scaled by a factor of 2,  $\sigma_{\text{FoG}}^2 \propto M^{2/3}$  would change by  $11 \text{ Mpc}^2$ .

Given the above considerations, we adopted a generous hard prior on  $\sigma_{\text{FoG}}^2$  between 0 and  $40 \text{ Mpc}^2$ .

## APPENDIX C: MARKOV CHAIN MONTE CARLO (MCMC) METHODS

We adopt a hybrid MCMC/importance resampling approach to explore the BOSS likelihood surface with various priors imposed from the WMAP7 likelihood in Models 1–4. This approach is necessary in our case because our model evaluation is slow, and we must marginalize over  $b\sigma_8$  and  $\sigma_{\text{FoG}}^2$  at each point in parameter space that we consider. This section describes our methods in each case.

### C1 Importance resampling

WMAP7 MCMC chains are publicly available<sup>5</sup>. These chains provide a fair sample of the WMAP7 likelihood surface. Importance resampling (Gilks, Richardson & Spiegelhalter 1996; Lewis & Bridle 2002) allows us to compute how constraints on the model parameters change given an additional constraint by evaluating the new likelihood at a subsample of the original MCMC chain, multiplying the original weight of each element by the new likelihood, and then recomputing confidence intervals.

### C2 Model 1: WMAP7+CMASS $\Lambda\text{CDM}$

For this model we marginalize only over two parameters, so it is feasible to directly compute a marginalized CMASS likelihood:

$$P_{\text{CMASS}}(\theta_{\Lambda\text{CDM}}) = \int db\sigma_8 d\sigma_{\text{FoG}}^2 e^{-\chi_{\text{CMASS}}^2(\theta_{\Lambda\text{CDM}}, b\sigma_8, \sigma_{\text{FoG}}^2)/2}. \quad (\text{C1})$$

We then use importance resampling of the WMAP7 chain parameters.

### C3 Model 2 WMAP7+CMASS $\Lambda\text{CDM}$ geometry, free growth; Model 3 WMAP7+CMASS $\Lambda\text{CDM}$ growth, free geometry

In Models 2 and 3, for each linear matter power spectrum, we must vary three or four extra parameters describing the galaxy clustering observations. We therefore explore the likelihood surface at each point in the WMAP7 chain by MCMC, and thereby sample the CMASS likelihood distribution  $P(\theta_{\text{CMASS}}|\theta_{\Lambda\text{CDM}})$ . In Model 2,  $\theta_{\text{CMASS}} = \{b\sigma_8, \sigma_{\text{FoG}}^2, f\sigma_8\}$ , and in Model 3,  $\theta_{\text{CMASS}} = \{b\sigma_8, \sigma_{\text{FoG}}^2, D_V, F\}$ . MCMC chains at a fixed  $\theta_{\Lambda\text{CDM}}$  sample parameter space proportional to  $P(\theta_{\text{CMASS}}|\theta_{\Lambda\text{CDM}})$ , but in order to compute the marginalized likelihood of  $f\sigma_8$  in Model 2 or  $D_V, F$  in Model 3, we must determine the relative likelihood of the MCMC chains evaluated at different  $\theta_{\Lambda\text{CDM}}$ . Since

$$Ne^{-\chi^2(\theta_{\text{CMASS}}, \theta_{\Lambda\text{CDM}})/2} = P(\theta_{\text{CMASS}}, \theta_{\Lambda\text{CDM}}) = P(\theta_{\text{CMASS}}|\theta_{\Lambda\text{CDM}})P(\theta_{\Lambda\text{CDM}}) \quad (\text{C2})$$

where  $N$  is an overall normalization, we can combine the  $\chi^2$  computed at any point with the probability density estimated by our MCMC to determine the relative normalization of  $P(\theta_{\Lambda\text{CDM}})$ . In practice, we first normalize each MCMC distribution so that  $\int P(\theta_{\text{CMASS}}|\theta_{\Lambda\text{CDM}}) = 1$ , find  $\theta_{\text{CMASS}}^*(\theta_{\Lambda\text{CDM}})$  with the minimum value of  $\chi^2$  in each chain, and integrate  $P(\theta_{\text{CMASS}}|\theta_{\Lambda\text{CDM}})$  in a small, fixed size region of parameter space around  $\theta_{\text{CMASS}}^*$ , which we

<sup>5</sup> <http://lambda.gsfc.nasa.gov/product/map/dr4/parameters.cfm>

call  $\tilde{p}(\theta_{\text{CMASS}}^*, d\theta_{\text{CMASS}})$ .<sup>6</sup> The relative weight of each point in the WMAP-only MCMC chain is then determined by the CMASS-only likelihood value and  $\tilde{p}$ :

$$w(\theta_{\text{ACDM}}) = e^{-\chi^2(\theta_{\text{CMASS}}^*|\theta_{\text{ACDM}})/2} (\tilde{p}(\theta_{\text{CMASS}}^*, d\theta_{\text{CMASS}}))^{-1}. \quad (\text{C3})$$

We find that our constraints are the same if we neglect this volume weighting factor  $\tilde{p}^{-1}$ , indicating that the effective volume of parameter space allowed by the CMASS measurements does not strongly depend on the underlying cosmological parameters  $\theta_{\text{ACDM}}$  when exploring the region of this parameter space allowed by WMAP7.

#### C4 Model 4 WMAP7+CMASS, free growth, free geometry

In this case, we use WMAP data to provide a prior on the shape of the linear matter power spectrum, which is well approximated by a multivariate Gaussian in the parameters  $\Omega_c h^2$ ,  $\Omega_b h^2$ ,  $n_s$ . WMAP7 constraints on these parameters primarily come from ratios of peak heights and the overall shape, rather than the locations of the peaks (which are sensitive to the angular diameter distance to the last scattering surface). Therefore, marginalized likelihood for these parameters is nearly independent of the adopted model for the low redshift expansion history (i.e., whether  $\Omega_k$  or  $w$  are freely varied). Thus we are able to make use of the CMB information on the underlying linear matter power spectrum that is independent of the model for the low redshift expansion history, and does not contain information on the distance to the last scattering surface. By using the linear matter power spectrum as a “standard ruler”, we are able to infer information about the geometric parameters  $D_V(z_{\text{eff}})$  and  $F$ . In this case we run an MCMC chain with the following 8 parameters, adding an additional multivariate Gaussian likelihood representing the CMB prior on the three linear matter power spectrum parameters:  $\{\Omega_c h^2, \Omega_b h^2, n_s, D_V(z_{\text{eff}}), F, b\sigma_8, f\sigma_8, \sigma_{\text{FoG}}^2\}$ .

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<sup>6</sup> A further subtlety is that to compute  $\tilde{p}$ , we restrict  $\theta^*$  to be sufficiently far from the hard prior boundary of  $\sigma_{\text{FoG}}^2$ , so that the volume element  $d\theta_{\text{CMASS}}$  does not intersect the hard boundary. The difference between  $\chi^2(\theta^*)$  and the global  $\chi^2$  minimum is small, since  $\sigma_{\text{FoG}}^2$  is poorly constrained by our measurements.

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